MA 125-5B, Spring 2004

# TEST # 4

April 23, 2004 (70 minutes)

Name: SSN:

Max. Points: 100 + 5 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they are needed. You may write on the back of a page if you need extra space.

**No book, no notes, and no calculator are to be used!**

**1.** For the following functions find the most general antiderivative (5P+5P):

(a) $f(x) = \dfrac{1}{x^2+1} - 3e^x$

(b) $f(t) = \dfrac{t - t^{5/2}}{t^2}$

**2.** Evaluate the definite integrals ($5 \times 5$P):

(a) $\displaystyle\int_{-1}^{1} (x^2 - 2x)\, dx$

(b) $\displaystyle\int_{0}^{\pi/4} \frac{2}{\cos^2 x}\, dx$

(c) $\displaystyle\int_{0}^{1} 3^x\, dx$

(d) $\displaystyle\int_{-1}^{2} |x|\, dx$

(e) $\int_0^{\pi/4} \cos(2x)\,dx$

**3.** (a) Evaluate the Riemann sum for $f(x) = 9 - x^2$ on the interval $[0, 3]$. Use three subintervals and take right endpoints as sample points. (8P)

(b) Is your result from part (a) an overestimate or an underestimate for $\int_0^3 (9 - x^2)\,dx$? Why? (3P)

**4.** Suppose that $\int_0^2 f(x)\,dx = -3$, $\int_7^{10} f(x)\,dx = 2$ and $\int_2^{10} f(x)\,dx = 5$. Find $\int_0^{10} f(x)\,dx$ and $\int_2^7 f(x)\,dx$. (8P)

**5.** The graph of $f$ is shown. Find $\int_{-1}^{1} f(x)\,dx$ and $\int_{-2}^{2} f(x)\,dx$. (8P)

**6.** (a) State the Evaluation Theorem. (5P)

(b) State the Fundamental Theorem of Calculus (Part 1). (5P)

**7.** A particle moves along a straight line with acceleration function $a(t) = t - 2$, measured in m/s$^2$. Its initial velocity is $v(0) = 0$ m/s and its initial position is 10 m.

(a) Find its position function $s(t)$ after $t$ seconds. (6P)

(b) Find its displacement in the time interval $0 \leq t \leq 3$. (3P)

(c) Find the total distance travelled by the particle in the interval $0 \leq t \leq 3$. (6P)

**8.** Order the following three numbers by their size (i.e. which one is smallest, which is largest and which is in the middle) and justify the order (8P):

$$2, \quad \int_0^2 \sqrt{1+x^2}\,dx, \quad \int_0^1 \sqrt{4-x}\,dx$$

**9.** Calculate (5P+5P$^*$)

(a) $\dfrac{d}{dx}\displaystyle\int_1^x \sin(t^2)\,dt$

(b)$^*$ $\dfrac{d}{dx}\displaystyle\int_{x^3}^1 \sin(t^2)\,dt$