TEST # 1

February 6, 2004 (70 minutes)

Name:	SSN:	
Max. Points: 100 + 5 Bonus	Points:	Test Grade:
Turn in all the work which you did to solve the problems, not just the final answer. In particular, include intermediate steps in calculations wherever they are needed. You may write on the back of a page if you need extra space.		
To receive credit, all solutions have to be based on the methods from Chapter 2 of Stewart's book.		
No book, no notes, and no calculator are to be used!		
1. (a) Define what it means that a function f is continuous at a number a . (3P) (b) Define what it means that a function f is differentiable at a number a . (3P)		
2. State the intermediate value theorem. (6P)		

3. Find the following limits: (4P+4P+4P+4P)

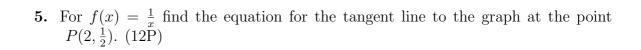
(a)
$$\lim_{x \to -2} \frac{x+2}{x^2-4}$$

(b)
$$\lim_{x \to 0} \frac{1}{\cos x}$$

(c)
$$\lim_{x \to \infty} \frac{2x^3 - x}{1 - 3x^3}$$

(d)
$$\lim_{x \to 0} \frac{1}{x^3}$$

- 4. From the graph of f provided on the right find all numbers a such that (3P+3P+3P+3P)
 - (a) $\lim_{x\to a^+} f(x)$ does not exist
 - (b) $\lim_{x\to a} f(x)$ does not exist
 - (c) f is not continuous at a
 - (d) f is not differentiable at a



6. The position of a vehicle after t seconds is $s(t) = t^3$ meters. (a) Find the velocity and the acceleration of the vehicle after 2 seconds. (12P)

(b) At which time does the vehicle have a velocity of 27 m/s? (4P)

7. For the given graph of f, sketch the graphs of f' and f''. Make sure that these graphs reflect where f is increasing or decreasing and where f is concave upwards or concave downwards. (14P)

8. The following is known about the derivatives f' and f'' of a function f:

$$f'(x)>0$$
 on $(-\infty,-1)$ and $(1,\infty)$

$$f'(x) < 0$$
 on $(-1, 1)$

$$f'(-1) = 0, f'(1) = 0$$

$$f''(x) < 0 \text{ on } (-\infty, 0)$$

$$f''(x) > 0 \text{ on } (0, \infty)$$

- (a) Where does the graph of f have horizontal tangents? (3P)
- (b) Where does the graph of f have inflection points? (3P)
- (c) Sketch a possible graph of f. (12P)

(d)* How many different graphs are possible for f and how do they differ? (5P Bonus)