MA 125-5B, Spring 2004

TEST # 1

February 6, 2004 (70 minutes)

Name: SSN:

Max. Points: 100 + 5 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations** wherever they are needed. You may write on the back of a page if you need extra space.

To receive credit, all solutions have to be based on the **methods from Chapter 2** of Stewart's book.

No book, no notes, and no calculator are to be used!

1. (a) Define what it means that a function f is continuous at a number a. (3P)

(b) Define what it means that a function f is differentiable at a number a. (3P)

2. State the intermediate value theorem. (6P)

3. Find the following limits: (4P+4P+4P+4P)

(a) $\lim_{x \to -2} \dfrac{x+2}{x^2-4}$

(b) $\lim_{x \to 0} \dfrac{1}{\cos x}$

(c) $\lim_{x \to \infty} \dfrac{2x^3 - x}{1 - 3x^3}$

(d) $\lim_{x \to 0} \dfrac{1}{x^3}$

4. From the graph of f provided on the right find all numbers a such that (3P+3P+3P+3P)

(a) $\lim_{x \to a^+} f(x)$ does not exist

(b) $\lim_{x \to a} f(x)$ does not exist

(c) f is not continuous at a

(d) f is not differentiable at a

5. For $f(x) = \frac{1}{x}$ find the equation for the tangent line to the graph at the point $P(2, \frac{1}{2})$. (12P)

6. The position of a vehicle after t seconds is $s(t) = t^3$ meters. (a) Find the velocity and the acceleration of the vehicle after 2 seconds. (12P)

(b) At which time does the vehicle have a velocity of 27 m/s? (4P)

7. For the given graph of f, sketch the graphs of f' and f''. Make sure that these graphs reflect where f is increasing or decreasing and where f is concave upwards or concave downwards. (14P)

8. The following is known about the derivatives f' and f'' of a function f:

$f'(x) > 0$ on $(-\infty, -1)$ and $(1, \infty)$

$f'(x) < 0$ on $(-1, 1)$

$f'(-1) = 0$, $f'(1) = 0$

$f''(x) < 0$ on $(-\infty, 0)$

$f''(x) > 0$ on $(0, \infty)$

(a) Where does the graph of f have horizontal tangents? (3P)

(b) Where does the graph of f have inflection points? (3P)

(c) Sketch a possible graph of f. (12P)

(d)* How many different graphs are possible for f and how do they differ? (5P Bonus)