

Mathematics 125 **Midterm 2**
March 18, 2004

- Calculators are allowed *only* for numerical calculations, that is you may not graph functions on your calculator.
- There are a sheet of scratch paper attached at the end of the exam. Use it but do not tear them off the exam.
- Show your work; clearly write down each step in your calculations/reasonings. *No credit* is given to a correct numerical answer *without* any justification.

1.(55 pts) Differentiate the following functions.

a)

$$\frac{x^2 - x}{\sqrt{x}}$$

b)

$$\left(1 - \frac{1}{x}\right)^3$$

c)

$$e^{\sin x^2}$$

d)

$$e^{e^x}$$

e)

$$\sin(x^2 + 1) \cos(x^3 + 1)$$

f)

$$(2x + 1)^3 (x^4 + 1)^2$$

Hint: logarithmic differentiation

g)

$$\tan^3(x^3 + x^2)$$

h)

$$\arctan(x^3 + x^2)$$

Hint: $y = \arctan(x^3 + x^2)$ is equivalent to $\tan y = x^3 + x^2$. Implicitly differentiate.

i)

$$\ln x$$

Hint: *Yes*, the answer is $1/x$! You are to derive it from scratch using implicit differentiation. Look at the hint for **h**.

2.(15 pts) **a)** Draw the set of points satisfying $x^2 + y^2 = 25$ and plot the point $(4, 3)$ on it.

b) Find an equation of the tangent line to the set $x^2 + y^2 = 25$ at $(4, 3)$

3. (15 pts) As a baloon leaks air, the area of the baloon decreases at $5 \text{ cm}^2/\text{min}$. Find the rate at which the radius of the baloon decreases when the radius of the baloon is 20 cm.
Formuli you may need: volume of a sphere of radius r : $\frac{4\pi}{3}r^3$ and area of a sphere of radius r : $4\pi r^2$.

4. (15 pts) Suppose $f(x)$ is the number of pounds a company sells coffee beans when it is sold at x dollars per pound. Then the total revenue the company collects is $T(x) = xf(x)$.
- a) When $f(6) = 20,000$ and $f'(6) = -3000$, what is $T'(6)$?

- b) Using linear approximation, approximate the value $T(7)$. In particular, was raising the price from 6 dollars/pound to 7 dollars/pound good for the business? Explain.