Calculus III Final Exam. April 2003. Name___________________________________

Mathematically justify your answers (show work). Simplify and complete all computations as much as possible. Circle answers.

1. Near the surface of the planet X-12 a falling body undergoes a downward acceleration of 5m/sec^2. A projectile is fired on X-12 from ground level with an initial speed of 300m/s at a 30° angle of elevation above the horizontal. Ignoring friction, find (A) the position $\overrightarrow{r}(t)$ at time t (before it strikes the ground), and (B) the range of the projectile.

2. The position of a particle at time t is given by $\overrightarrow{r}(t) = \cos(3t)\overrightarrow{i} + \sin(3t)\overrightarrow{j} + t\overrightarrow{k}$. Find a parametric equation for the line tangent to the path when $t = \pi/4$.

3. Let $f(x, y) = \tan^{-1}(\frac{y}{x})$. Find (and **simplify**)

(A) f_x

(B) f_y

(C) f_{xy}

4. Find an equation for the plane tangent to the surface $x^2 + 3y^2 + 5z^2 = 9$ at the point $(x, y, z) = (-1, 1, -1)$.

5. Express in two different iterated integrals $\iint_D f(x, y)d$ if D is the region bounded by $y = 3x$ and $y = x^2$.

6. Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^3 + y^3 + z^3$ on the sphere $x^2 + y^2 + z^2 = 1$.

7. Let $f(x, y) = 4xy - \frac{1}{2}x^2 - y^4$. Find (and LIST) all critical points and use the second derivative test to identify all local minima, maxima, and saddle points.

8. Use polar coordinates and integration to find the volume of the region inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

9. Let a vector field be $\overrightarrow{F}(x, y) = y\overrightarrow{i} - x\overrightarrow{j}$. Find the work done if a unit mass in the field traverses once in the counterclockwise direction around the circle $x^2 + y^2 = 9$.

10. Suppose the force exerted on a unit mass at the point (x, y) is $\overrightarrow{F}(x, y) = (3x^2 - y^2 - 4)\overrightarrow{i} + (3y^2 - 2xy + 1)\overrightarrow{j}$.

(A) Show the field is conservative and find its potential.

(B) Find the work done in moving the mass from $(0, 0)$ along straight line to $(3, 19)$ and then along another line to $(1, 2)$.

11. Use Green's Theorem to show that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab. Explain in a coherent manner your reasoning. (Hint: let $x = a\cos t$, $y = b\sin t$).

12. Let $w = g(x, y, z)$ be continuous and have continuous first and second derivatives. Show that **curl**$(\nabla g(x, y, z)) = 0$.

13. Let a vector field be $\overrightarrow{F}(x, y, z) = (x - y + 3z)\overrightarrow{i} + (y - x - 3z)\overrightarrow{j} + (3x - 3y + 9z)\overrightarrow{k}$.

Show that $\overrightarrow{F}$ is a conservative field and find its potential.

14. Use cylindrical coordinates and integrals to show that the volume of a right circular cylinder of radius R and altitude H is $\frac{1}{3}\pi R^2 H$.

Surprise Extra Credit: Two problems. You may do both.

(1.) Use the method of Lagrange multipliers to find the minimum and maximum values of $f(x, y, z) = xyz$ on the ellipsoidal surface $\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1$.

(2) Use the result of problem 11 above to show that the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{4}{3}\pi abc$. (Hint: Sum the horizontal slices through the ellipsoid) (You need not have done #11—use the result)..