Calculus III Test 1 Jan. 30, 2003 NAME______________________________________

No calculators, books, or notes allowed. Justify your answers by giving appropriate arguments and steps. Circle answers. All problems will be of equal value. Be sure to work the given problem; otherwise you will not receive credit.

1. Let the curve C be given by $\overrightarrow{r}(t) = \langle t^2, 1-t^3, 1+t^3 \rangle$. Find parametric equations for the tangent line to the curve at the point $\overrightarrow{r}(2) = \langle 4, -7, 9 \rangle$.

2. Find the length of the curve $\overrightarrow{\mathbf{r}}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + 2t^{3/2}\mathbf{k}$ for $0 \le t \le 7$.

3. Consider an ellipse $x^2/4 + y^2/9 = 1$. Find its curvature at the vertices $(2,0)$ and $(0,3)$. (Hint: Try letting $x = 2\cos t$, $y = 3\sin t$.)

4. Find the velocity, speed, and acceleration at time t for a particle with position given by $\overrightarrow{\mathbf{r}}(t) = (\cos 2t)\mathbf{i} - (\sin 2t)\mathbf{j} + t\mathbf{k}$.

5. There is a battle on the planet Xix. A projectile is fired from ground level with an initial speed of $200m/s$ at a $30°$ angle of elevation above the horizontal. Ignoring friction, find (horizontal) the range of the projectile (Important: On Xix downward acceleration due to gravity is $-5m/s^2$).

6. Let $f(x,y) = x\sin(3x+5y)$. Find the first partial derivatives of f.

7. Let $f(x,y) = y/(x^2+1)$. Sketch and label the level curves given by $f(x,y) = k$ for $k = -2, -1, 0, 1, 2$.

8. (10) Suppose $z = g(\frac{y}{x})$. Assume g has continuous first and second derivatives. Find $\partial z/\partial x$ and $\partial z/\partial y$.

9.Determine which of the following functions solve the partial differential equation $u_{xx} + u_{yy} = 0$.

(A) $u = \tan^{-1}(\frac{y}{x})$

(B) $u = \ln(x^2+y^2)$

10. Let $f(x,y) = xy$. (A) Sketch and label the level curves for $f(x,y) = 1$ and $f(x,y) = -4$. (B) Let $\overrightarrow{V}(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$ and sketch representations of $\overrightarrow{V}(x,y)$ with tail at the point (x,y) on each of the two level curves with x values given by $x = \pm 1, \pm 2$. (C) What can you conclude about the relation between the vectors $\overrightarrow{V}(x,y)$ and the level curves?

11. Do one of the following: (Circle the letter of the problem you attempt).

(A) Find the following limit, or show it does not exist: $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$

(B) Find the unit tangent $\overrightarrow{T}$ and unit normal $\overrightarrow{N}$ at each point on $\overrightarrow{r}(t) = \langle t, t^2, t^3 \rangle$.

Extra Credit: The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure P, and volume V is $PV = mRT$, where R is the gas constant. Show that $\frac{\partial P}{\nabla V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$.