MA 227-5D Spring 2003 Test 1
Name ___________________________________

1. Consider the curve C determined by the position function

$$\mathbf{r}(t) = t\mathbf{i} + \cos(\pi t)\mathbf{j} + \sin(\pi t)\mathbf{k}$$

for $t \geq 0$.
(a) Find the unit tangent to C at $(1, -1, 0)$.
(b) Determine $s(t)$, the length of the curve from $(1, -1, 0)$ to $(t, \cos(\pi t), \sin(\pi t))$.
(c) Parametrize C in terms of arc length.
(d) Determine the curvature κ of C at $(2, 1, 0)$.
2. Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} \text{ for } (x, y) \neq (0, 0) \\ 0 \text{ for } (x, y) = (0, 0) \end{cases}.$$

Determine whether f is continuous at the origin.
3. Let $z = f(x, y)$ and let $x = s - t$, $y = s + t$. Show that

$$\left(\frac{\partial z}{\partial y}\right)^2 - \left(\frac{\partial z}{\partial x}\right)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}.$$

4. Let $f(x, y) = x^2 = y\cos(\pi x) + e^{x-y^2}$. Consider the surface $\sum$ defined by $z = f(x, y)$.
(a) Find the equation of the tangent plane to $\sum$ at $(1, 1, 3)$.
(b) Determine the linearization of $f(x, y)$ at $(1, 1)$.
(c) In what direction at $(1, 1)$ does the function have the largest rate of change, and what is the magnitude of this largest rate of change?
(d) Determine the rate of change of $f(x, y)$ at $(1, 1)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$.
(e) Find a normal vector to $\sum$ at $(1, 1, 3)$.
(f) Find the equation of the normal line to $\sum$ at $(1, 1, 3)$.