Calculus II, Test 1

May 16, 2003

1. Find the antiderivatives:
a) $\int x \ln x dx$
b) $\int \frac{x^2}{1+x} dx$
c) $\int \frac{1}{x^2-9} dx$
d) $\int e^{3x} \sin x dx$
e) $\int x \cos 2x dx$
f) $\int \cos^4(x)\, dx$
g) $\int \cos^3(x)\, dx$.
2. Find the following integrals (show all details)
a) $\int_0^2 \frac{x^3}{1+x^4} dx$,
b) $\int_0^1 \frac{1}{9+x^2} dx$
c) $\int_0^\infty x e^{-x} dx$
3. Does $\int_1^\infty \sqrt{x} e^{-x^2} dx$ converge? Show why or why not.
4. Define the definite integral from a to b of f.
5. Evaluate (give an actual number)

$$\lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^{N} \left(1 + \frac{2i}{N}\right)^3.$$

6. Find the area between $y = x^2$ and $y = x^3$.
7. Rotate the region between $y = x^2$ and $y = x^3$ about the y axis. What is the volume?
8. Rotate the region between $y = x^3 + 4x$, the y axis, the x axis, and $x = 1$ about the y axis. Find the volume.
9. a) Show that $\int_0^\infty e^{-x^5} dx$ converges.
b) Show that the above integral equals

$$\int_0^1 (\ln x)^{\frac{1}{5}}\, dx.$$