Test 1

JANUARY 24, 2003

Calculus I Name _____

You may not use calculators, notes, or books. Do your own work.

Justify your answers mathematically. 'Show your work.' **CIRCLE ANSWERS.**

A. In each of the following, find the limit, if it exists. Infinite limits are allowed. If a limit fails to exist, so state [4 each].

1. $\lim_{x\to 2}(x^2 - 3x + 2)$
2. $\lim_{y\to 2} \frac{y^2-7y+10}{y-2}$
3. $\lim_{t\to 3} \frac{t-3}{t^2-6t+9}$
4. $\lim_{x\to\infty} \frac{3-5x+5x^2}{7x^2+4x+2}$
5. $\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3}$
6. $\lim_{h\to 0} \frac{(2+h)^3-8}{h}$
7. $\lim_{x\to 5^+} \frac{1-x}{x-5}$
8. $\lim_{x\to 3^-} \frac{x^2-9}{|x-3|}$
9. Find all horizontal and vertical asymptotes and sketch the graph.of $y = \frac{x+3}{x-2}$.
10. Use the definition of derivative to find $f'(a)$ for $f(x) = x^3$.
11. Use the definition of derivative to find $f'(a)$ for $f(x) = \frac{1}{x}$.
12. Let $g(x) = \sqrt{x}$. (a) Find the slope of the line tangent to the graph of $y = g(x)$ at $x = 9$. (b) Find the equation of the line tangent to the graph of $y = g(x)$ at the point $(9, 3)$.
13. Suppose a particle moves along a line and its position at time t is given by $s(t) = 3t-2t^2$. Find (a) the average velocity from $t = 1$ to $t = 3$. (b) Use limits to find the instantaneous velocity at $t = 3$.
14. Sketch the graph of a continuous function $y = g(x)$ whose derivative $g'(a)$ has the following properties: $g'(a) > 0$ for all $a < 3$, $g'(a) < 0$ for $3 < a < 6$, and $g'(a) > 0$ for $a > 6$.
15. $f(x) = \frac{x}{x^2-1}$

D.[6] Use the difference quotient and limit to find the following:

$f'(3)$ if $f(x) = x^2$.

E.[6 each]Use differentiation rules to find $f'(x)$, and simplify answers, if:

16. $f(x) = 3x^7 + 5x^5 - \sqrt{3}x + 12$
17. $f(x) = \frac{5x+3}{3x-1}$
18. $f(x) = \frac{1}{x} + \frac{x}{3}$
19. $f(x) = \frac{x^3}{1-x^4}$

F.[6 each] Work the following:

20. Suppose a, b, c are constants and N is a positive integer. If $z = ay^N + by + c$, find $\frac{dz}{dy}$.
21. Let $y = 2x^4 - 5x^3 + x^2 - 8x + \sqrt{32.78}$. Find $y'' = \frac{d^2y}{dx^2}$.
22. Find the equation of the line tangent to $y = x - 2x^3$ at $(x, y) = (1, -1)$.
23. Find all points on the curve $y = x^3 - 12x + 10$ where the tangent is parallel to the x-axis.

G. [10] Suppose a body moves upward in a vertical line under gravity alone (air resistance, etc., neglected) with initial velocity of 64 ft/sec. Using the calculus methods as discussed in class, find:

(a.) An expression for its height s (in ft) at time t;

(b.) the length of time it rises;

(c.) how high it goes.