MA 125-8C, Spring 2003

FINAL EXAM

April 29, 2003 (150 minutes)

Name: SSN:

Max. Points: 100 + 8 Bonus Points: Exam Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they demonstrate which method you used to get the result. You may use separate sheets for this.

The exam is **closed book** and **closed notes**. You may use a calculator.

1. Find the following limits ($4 + 4 + 4^*$ pts):

(a) $\lim_{x\to\infty} \dfrac{x - x^2}{1 + 3x^2}$

(b) $\lim_{x\to 0} \dfrac{x^2}{x - \sin x}$

(c)* $\lim_{x \to 0^+} x(\ln x)^2$

2. Use the definition of the derivative to find $f'(x)$ for the function $f(x) = \frac{1}{x}$ (5 pts).

3. (a) Find the linearization $L(x)$ of $f(x) = \cos x$ at $a = \pi/2$ (4 pts).

(b) Use $L(x)$ to give an approximation for $\cos 1.6$ and $\cos 2.5$. Which of those two approximations should be close to the correct value and why? (4 pts)

4. The graph of the derivative f' of a function f is provided. Use it to find the following:

(a) intervals of increase and decrease for f (2 pts),

(b) local minima and local maxima of f (2 pts),

(c) intervals where the graph of f is concave upwards or concave downwards (2 pts),

(d) inflection points of f (2 pts).

(e) Sketch the graph of f'' (2 pts).

(f) Assume that $f(0) = 0$. Sketch the graph of f (4 pts).

5. Find the derivative $f'(x)$ for the following functions $(4 + 4 + 4 + 4^*$ pts):

(a) $f(x) = xe^{x^2}$

(b) $f(x) = \sin(3x) - \cos(4x)$

(c) $f(x) = \dfrac{e^x + 1}{e^x - 1}$

(d)* $f(x) = x^{\ln x}$

6. (a) For the implicit function $x^2y = e^x + y^3$ find the derivative y' in terms of x and y. (5 pts)

(b) Find an equation for the tangent to the graph of the above implicit function at the point $(x, y) = (0, -1)$. (3 pts)

7. Find all critical numbers as well as the absolute maximum and absolute minimum of $f(x) = x + \cos x$ on the domain $-\pi \leq x \leq \pi$. (8 pts)

8. A closed top box with square base is to be built with a surface area of $600\,\text{cm}^2$. How should the height and base length of the box be chosen to get a box of largest possible volume? (8 pts)

9. State the Mean Value Theorem and provide a sketch which explains its meaning. (5 pts)

10. Let $f(x) = x^3$.

(a) Find the definite integral of f from $x = -1$ to $x = 1$. (4 pts)

(b) Find the total area between the graph of f and the x-axis from $x = -1$ to $x = 1$. (4 pts)

11. Calculate: $(4 + 4$ pts)

(a) $\displaystyle\int_1^4 \frac{1}{x\sqrt{x}}\,dx$

(b) $\displaystyle\int_0^1 \frac{2}{\sqrt{1-x^2}}\,dx$

12. The acceleration (in m/sec^2) of a particle which moves along a straight line is given by $a(t) = 2t + 1$. It is also know that its velocity (in m/sec) and position (in meters) at time $t = 0$ are $v(0) = 1$ and $s(0) = -1$. Find the position function $s(t)$ for all times t. (8 pts)