MA 125-8C, Spring 2003

TEST # 2

March 13, 2003 (105 minutes)

Name: SSN:

Max. Points: 100 + 10 Bonus Points: Test Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they are needed. You may use separate sheets for this.

The test is **closed book** and **closed notes**. You may use a calculator.

1. Find $f'(x)$ for the following functions ($6 \times 5 = 30$ points):

(a) $f(x) = 3x^3 + \ln x - \dfrac{2}{\sqrt{x}}$

(b) $f(x) = \dfrac{x^2 - x}{x^3 - 1}$

(c) $f(x) = 2^x \tan^{-1} x$

(d) $f(x) = e^{\cos x} - \cos e^x$

(e) $f(x) = \sin \sin \sin x$

(f) $f(x) = x^{\sin x}$

2. Suppose that $F(x) = f(g(x))$ and that $f(1) = 5$, $g(1) = 2$, $g'(1) = -3$ and $f'(2) = 4$. Find $F'(1)$. (5 points)

3. For the function $g(x) = 2x + \sin x$ with domain $0 \leq x \leq 2\pi$ find the intervals where the graph of g is (i) increasing, (ii) decreasing, (iii) concave upwards, and (iv) concave downwards. (15 points)

4. Find the equation for the tangent line at $(\pi/2, 0)$ to the graph of the implicit function (15 points)

$$y \sin x + x \cos y = x.$$

5. Find the linearization of the function $f(x) = \sqrt[3]{x}$ at $a = 8$. Use your result to find an approximation for the value of $\sqrt[3]{8.05}$. (10 points)

6. (a) Explain by a sketch and in words how Newton's Method is used to find a better approximation x_2 of a root of a function $f(x)$ if an initial (rough) approximation x_1 is given. (10 points)

(b) Use Newton's Method to find an approximation x_2 for a solution of $x^6 = 2$ if $x_1 = 1$ is used as initial approximation. (5 points)

7. Car A is leaving from an intersection of two roads at 1:00pm, driving north at a constant speed of 45 mi/h. Car B is leaving from the same intersection at 2:00pm, driving east at 60 mi/h. At what rate is the distance of the two cars increasing at 4:00pm? (10 points)

8. A pool is 15 feet wide and 30 feet long. Its depth varies between 3ft and 6ft according to the lengthwise cross section sketched in the figure. The pool is filled at a rate of 50 ft^3/hour. How fast is the water level rising when the depth at the deep end of the pool is 2 feet? (10 points bonus)