EGR 265-6D, Math Tools for Engineering Problem Solving

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Number	Score	Max Score
1		8
2		8
3		14
4		12
5		10
6		8
7		8
8		12
9		12
10		8
Total		100

* For graders only

EGR 265-6D, Math Tools for Engineering Problem Solving
December 9, 2011, 1:30pm to 4:00pm

Name (Print last name first): .....

Student ID Number: ....

Final Exam

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$3y^2y' = 2x(y^3 + 1), \quad y(0) = 0.$$

Problem 2 (8 points)

A population of bacteria decays proportional to the number of bacteria present at time t due to the effect of certain medicine. Suppose that the initial population is $20,000$ and that the population after 2 hours has decreased to $15,000$.
(a) Find its decay rate k.
(b) What is the population after 4 hours?
(c) How long does it take the population to shrink to its half size?
Note: Write your answers in terms of natural logarithms, which do not need to be evaluated.

Problem 3 (14 points)

Consider the second order differential equation

$$y'' - 3y' + 2y = 2\cos(x) + \sin(x). \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 0$, $y'(0) = 1$.

Problem 4 (12 points)

A 1 m spring measures $\frac{13}{8}$ m long after a mass of 10 kg is attached to it. The medium through which the mass moves offers a damping force with damping coefficient $\beta = 80$ kg/s. Include the correct units in all your answers below.
(a) Find the spring constant k, assuming that $g = 10$ m/s^2.
(b) Find the equation of motion of the mass if it is released from a position 10 cm above the equilibrium position with a downward velocity of 40 cm/s (Choose the positive x-axis to be oriented downward).
(c) Will the mass return to the equilibrium position? If yes, when is the first time? If no, why not?

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = \dfrac{xy + 2}{y - x}$.
(b) Evaluate the directional derivative of $f(x, y)$ at the point $P(1, 2)$ in the direction of the vector $-\mathbf{i} + \mathbf{j}$.
(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $(1, 2)$. Also find the rate of increase in this direction.

Problem 6 (8 points)

Determine the equation of the normal line to the graph of $z = \dfrac{\ln x}{x+y} + 1$ through the point $(1, 0, 1)$.

Problem 7 (8 points)

Find the work done by the force field

$$\mathbf{F}(x, y) = y^2\mathbf{i} - e^x\mathbf{j}$$

along the curve given by the graph of $y = \frac{x^2}{2} - 1$, $0 \leq x \leq 1$.

Problem 8 (12 points)

(a) Verify that the force field $F(x,y) = (5x^4y^2 - y^6 + x)\mathbf{i} + (2x^5y - 6xy^5 - e^y)\mathbf{j}$ is conservative.
(b) Find a potential function $\phi(x,y)$ for $F(x,y)$.
(b) Find the work done by the force field $F(x,y)$ along the quarter circle $x = \cos(t)$, $y = \sin(t)$, $0 \leq t \leq \frac{\pi}{2}$.

Problem 9 (12 points)

A lamina of density $\rho(x, y) = x$ is bounded by the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.
(a) Find the lamina's moment of inertia I_y with respect to the y-axis.
(b) Find the lamina's moment of inertia I_x with respect to the x-axis.

Problem 10 (8 points)

Let R be the region in the **second** quadrant which lies between the circles of radius $r = \sqrt{2}$ and $r = \sqrt{5}$. Find

$$\int_R (1 + \sqrt{1 + x^2 + y^2})\, dA.$$