EGR 265, Math Tools for Engineering Problem Solving
November 30, 2011, 50 minutes

Name: .....

TEST III

Problem 1 (9+9 points)

(a) Let $f(x,y) = \frac{2x}{x-y}$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x,y) = x\ln(x^2+y)$ find g_x, g_y and g_{xy}.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = e^{(x^4+y^2)}$ find its direction **and** rate of steepest descent at the point $P(1, 2)$.

(b) Find the directional derivative of $h(x, y)$ at $P(1, 2)$ in the direction of the vector from $P(1, 2)$ to $Q(4, -2)$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of $z = xy + \cos(x - y) + 1$ at the point $(\frac{\pi}{2}, 0, 1)$.

(b) Also, find parametric equations for the normal line to the graph of $z = xy + \cos(x - y) + 1$ at $(\frac{\pi}{2}, 0, 1)$.

Problem 4 (12 points)

Evaluate $\int_C x^3 y\, ds$, where C is a quarter of a unit circle parametrized by $x = \cos(t)$, and $y = \sin(t)$, $0 \leq t \leq \frac{\pi}{2}$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = \frac{\ln(x)}{x}\mathbf{i} - x\sqrt{y}\mathbf{j}$$

along the curve C given by the graph of $y = x^2$, $1 \leq x \leq 2$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x,y) = y^2\sin(x)\mathbf{i} + \dfrac{y^3}{3}\cos(x)\mathbf{j}$

(b) $F(x,y) = (e^x + xy^2)\mathbf{i} + (x^2y + e^y)\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x,y)$ from Problem 6 find a potential function $\phi(x,y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = (1+t^2)\mathbf{i} + t\sqrt{t}\mathbf{j}$, $0 \le t \le 1$.

Scratch Paper

Scratch Paper