SPRING 2012 — MA 227 — FINAL EXAM
FRIDAY MAY 4, 2012

Name: ____

There are 14 questions, each worth 8 points; 100 (or more) points is equivalent to 100% for the exam. Partial credit is awarded where appropriate. Show all working; your solution must include enough detail to justify any conclusions you reach in answering the question.

1. Let $\mathbf{r}(t) = (t^2, t, t^4)$. Find normal plane at point $t = 1$.

2. Find the equation of the plane containing the points $(1, 2, 3)$, $(1, 1, -1)$ and $(-1, 2, 1)$.

3. Find the area of the parallelogram generated by the vectors $(2, 2, -1)$ and $(-1, 1, 3)$.

4. Let $f(x, y) = xe^y - x^2y^2$. Find all second partial derivatives: f''_{xx}, f''_{xy}, f''_{yy}.

5. Find local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 + 4xy + 6y^2 - 2y + 1.$$

6. Let $z = x^2y^2 + \frac{1}{y}$. Find equation of the tangent plane at point $(1, 1)$.

7. Find the maximum rate of change of $f(x, y) = y^2 - \frac{x}{y}$ at the point $(1, -1)$. In which direction does it occur?

8. Find the area of the region D bounded by $y = x^2$ and $y = 3x$.

9. Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_{x^4}^{x} f(x, y)dydx.$$

10. Find the volume under the surface $z = x^2 + y^2$ and above the ring $1 \leq x^2 + y^2 \leq 4$ in the xy plane.

11. Acceleration of the particle is given by $\mathbf{a} = (0, 1, 1)$. Find velocity and position of the particle as functions of time if at time $t = 0$ we have $\mathbf{v}(0) = (1, 1, -1)$ and $\mathbf{r}(0) = (0, -1, 1)$.

12. Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + 2y^2 - 4x + 1$ on the region $0 \leq x \leq 3, \quad -1 \leq y \leq 1$. Be sure to provide coordinates of the points and the values of absolute maximum and minimum.

13. Using spherical coordinates, calculate the integral $\int\int\int_V z\, dxdydz$, where the region V is the spherical layer in the first octant: $\{1 \leq x^2+y^2+z^2 \leq 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\}$.

14. Find the volume of the solid above the region $D = \{(x, y) : y^2 \leq x \leq 1\}$ in xy plane and below the surface $z = xy^2$.