MA 227, CALCULUS III

Fall, 2011

Name (Print last name first): .....

Student Signature: .....

TEST II

10 questions, 10 points each.
SHOW ALL YOUR WORK! CIRCLE YOUR ANSWER!

Question 1

Find the gradient of the function $f(x, y) = x^2 e^{-xy}$ at the point $(1, 0)$.

Question 2

Find the directional derivative of the function $f(x, y, z) = xz^2 + yz$ in the direction of the vector $\vec{v} = \vec{i} + 2\vec{j} - 2\vec{k}$ at the point $(2, 1, -1)$.

Question 3

Find local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 - 6xy + 4y^2 - 5y + 1.$$

Question 4

Let $z = xy^3 - \frac{1}{y}$. Find equation of the tangent plane at point $(-1, 1)$.

Question 5

Find linear approximation for the function

$$f(x, y) = x^2 - xy + y^3 x$$

near point $(2, 1)$.

Question 6

Let $f(x,y) = xy + \frac{x^2}{y}$ and $x = st, \quad y = s^2 + t$. Find partial derivatives $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

Question 7

Let $f(x,y) = y\sin(x) + x^2 y$. Find all second partial derivatives: $f''_{xx},\ f''_{xy},\ f''_{yy}$.

Question 8

Find equation of the tangent plane to the surface $x^3 - y^3 + z^2 = -8$ at the point $(-1, 2, 1)$.

Question 9

Find the maximum rate of change of $f(x, y) = y\sqrt{x} - \frac{x}{y}$ at the point $(1, -1)$. In which direction does it occur?

Question 10

Find the absolute maximum and absolute minimum of the function $f(x,y) = x^2+2y^2-2x+1$ on the region $-2 \leq x \leq 0, \quad -1 \leq y \leq 1$. Be sure to provide coordinates of the points and the values of absolute maximum and minimum.