

MA 126 CALCULUS II
 Wednesday, December 13, 2017

Name (Print last name first):

Student Signature:

Section:..... Instructor Name:

FINAL EXAM

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 13 questions. Clearly write your answer in the space provided after each question. You need to show some work to justify your answer for this part of the test. Limited partial credit is awarded for this part of the test. **CHECK YOUR ANSWERS!**

Each question is worth 4 points.

Question 1

A child is pulling a loaded toy-box on a level path with a force of 10 Newtons exerted at an angle of 30° above the horizontal. Find the horizontal and vertical components of the force.

Answer:

Question 2

Find the derivative of the function $f(x) = \tan^{-1}(\cot(x))$ for $0 < x < \pi$, and state the name of the differentiation rule you use. (Simplify the expression you obtain!)

Answer:

Question 3

Evaluate $\lim_{x \rightarrow 0} \frac{x}{x + \sin(x)}$.

Answer:

Question 4

Evaluate the indefinite integral $\int \sin(x) \cos^2(x) dx$.

Answer:

Question 5

Evaluate the indefinite integral $\int x e^{-2x} dx$.

Answer:

Question 6

Determine whether the improper integral $\int_0^{\infty} \frac{x}{1+x^4} dx$ is convergent or divergent.
Find its exact numerical value if it converges! (No approximation.)

Answer:

Question 7

Find the area of the region bounded by the parabola $y = x^2 + x$ and the line $y = -x$.

Answer:

Question 8

Use the method of cylindrical shells **to set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating about the y -axis the region bounded by the line $y = 2x$ and the parabola $y = 8x - x^2$.

Answer:

Question 9

Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(2x-1)^n}{\sqrt{n}}$. (Be sure to check any end-points!)

Answer:

Question 10

Determine whether the alternating series $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(n)}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific!)

Answer:

Question 11

Find an equation of the plane containing the point $P(0, 1, 1)$ and the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} + \mathbf{k}$.

Answer:

Question 12

Find the distance from the point $P(-1, 1, 1)$ to the plane $x + y + z = 1$.

Answer:

Question 13

Find the length of the curve with vector equation $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \frac{\sqrt{2}}{2}t^2, t \right\rangle$ for $1 \leq t \leq 2$.

Answer:

PART II

Each problem is worth 12 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers. CIRCLE YOUR ANSWER!

Problem 1

This problem has two separate questions. (Answer all the questions!)

(a) Find the **area** of the triangle with vertices $P(1, -1, 1)$, $Q(1, 1, 1)$ and $R(-2, 1, 2)$.

(b) The region bounded by the curves $y = \sqrt{x}$ and $y = x^3$ is rotated about the vertical line $x = -1$. Find the **volume** of the solid obtained in this way.

Problem 2

(a) Use the power series of $\frac{1}{1+x^4}$ to write the function $f(x) = \frac{x^2}{1+x^4}$ as a power series.

(b) Use the series in part (a) to find the minimum number of terms needed to approximate the integral

$$\int_0^{1/10} \frac{x^2}{1+x^4} dx$$

with an error less than 10^{-6} . (You do not need to compute and add up the terms in the sum!)

Problem 3

Find the work done in pumping all the water out of a cubic container with edge 4 m which is $3/4$ full (i.e., the container is three quarters full). The water has to be lifted all the way to the top of the cube in order to be removed. (You may use the approximation $g \approx 10\text{ m/s}^2$ and the water density $\rho = 1,000\text{ kg/m}^3$.)

Problem 4

Two planes are given by the equations $x - y - z = 1$ for the plane \mathcal{P}_1 and $x - y + z = 1$ for the plane \mathcal{P}_2 .

- (a) Determine whether or not these planes are parallel (justify your answer). If they are, find the distance between them. Otherwise, answer part (b) below.
- (b) Find both the parametric equations and the symmetric equations of the line of intersection of the planes \mathcal{P}_1 and \mathcal{P}_2 (if you have determined in part (a) that they are not parallel).

DO NOT ENTER ANY PROBLEM SOLUTIONS OR WORK ON THIS PAGE.

Summary of scores on problems - for grading purposes only.

	Points
Part I	
Questions 1 – 13	
Part II	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total Exam Score	

SCRATCH PAPER

(Scratch paper will not be graded)

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