# MA 125 OA, CALCULUS I

June 17, 2015

Name (Print last name first): .....

**TEST I**

**Show all your work! No partial credit will be given for the answer only!**

## PART I

**Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!**

**All problems in Part I are 7 points each**

Evaluate the following limits.

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = x^3$ is $f'(x) = 3x^2$.

Question 2

Find the derivative of $f(x) = x\cos(x)$

Answer: .....

Question 3

Find the derivative of $y = f(x) = \frac{1-x^2}{1+x^2}$.

Answer: .....

Question 4

Find the derivative of $y = f(x) = \sqrt[3]{x}(x + x^2)$.

Answer: .....

Question 5

Find the equation of the tangent line to the graph of $y = f(x) = \sin(x)$ at the point $a = \pi/3$.

Answer: .....

Question 6

Evaluate the limit $\lim_{\mathbf{x}\to\infty} \dfrac{\sin(x)}{x}$ (Note that $x \to \infty$!)

Answer: .....

Question 7

Evaluate the limit $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x^2 - 9}$

Answer: .....

**PART II**

**Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.**

## Problem 1 (10 points)

Suppose that $S(t) = t^5 - 5t$ $m$ is the position of a particle at time $t$ (in seconds) on a line. Find:
(a) the velocity at time $t$
(b) The displacement from $t = 0$ to $t = 2$
(c) the total distance traveled from $t = 0$ to $t = 2$.

## Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:

1. $\lim_{x \to -1^-} f(x) =$
2. $\lim_{x \to -1^+} f(x) =$
3. $\lim_{x \to -1} f(x) =$
4. $\lim_{x \to 2^-} f(x) =$
5. $\lim_{x \to 2^+} f(x) =$
6. $\lim_{x \to 2} f(x) =$
7. $\lim_{x \to \infty} f(x) =$
8. $\lim_{x \to -\infty} f(x) =$
9. State all intervals on which $f(x)$ is continuous.
10. State all intervals where $f(x)$ is differentiable.

## Problem 3 (10 points)

Find all points on the graph of $f(x) = x^3 + x$ where the tangent line is parallel to the line $y = 28x$.

## Problem 4 (11 points)

Suppose $P(x) = \frac{x}{x^2+1}$ describes the profit (in 100's of dollars) from selling $x$ (in thousands) items.

1. Given the definition of $P(x)$, what is the meaning of $P(2)$?

2. What is the meaning of $P'(2)$.

3. Compute $P'(x)$ for all $x \geq 0$. Determine where $P'(x) > 0$ and where $P'(x) < 0$.

4. Use the above information to make a rough graph of $P(x)$ and decide how many items you should sell to maximize the profit.

## Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. $\lim_{x\to\infty} \sqrt{x} + \sqrt{x+1}$

2. $\lim_{x\to\infty} \sqrt{x} - \sqrt{x+1}$

Scratch paper