**MA 126 CALCULUS II**
Wednesday, December 09, 2015

Name (Print last name first): .....

Student Signature: .....

Section: ..... Instructor Name: .....

**FINAL EXAM**

**Closed book - Calculators and One Index Card are allowed!**

**PART I**

**Part I consists of 10 questions. Clearly write your answer in the space provided after each question. You need to show some work to justify your answer for this part of the test. Limited partial credit is awarded for this part of the test. CHECK YOUR ANSWERS!**

Each question is worth 4 points.

Question 1

Find the derivative of the function $f(x) = \tan^{-1}(x^2)$, and state the name of the differentiation rule that you use!

Answer: .....

Question 2

Evaluate $\lim_{x \to 0^+} \dfrac{x^2}{x - \sin(x)}$.

Answer: .....

Question 3

Evaluate the indefinite integral $\int \sin^3(x)\cos(x)\,dx$.

Answer: .....

Question 4

Determine whether the improper integral $\int_1^\infty \frac{x^3}{2+x^5}\,dx$ is convergent or divergent.

Answer: .....

Question 5

Find the area of the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

Answer: .....

Question 6

Use the method of cylindrical shells **to set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating about the $y$-axis the region bounded by the vertical lines $x = 0$, $x = 1$, and the parabolas $y = x^2 + 1$ and $y = -x^2$.

Answer: .....

Question 7

Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n^2}$. (Check end-points as well!)

Answer: .....

Question 8

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+1}}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific!)

Answer: .....

Question 9

Find an equation of the plane containing both the point $P(1, 1, 0)$ and the vectors $\mathbf{a} = \langle 1, 0, 1\rangle$ and $\mathbf{b} = \langle 0, 1, 1\rangle$.

Answer: .....

Question 10

Find the length of the arc of the circular helix with vector equation $\mathbf{r}(t) = \langle 3\cos(t),\, 3\sin(t),\, 4t\rangle$ when $0 \leq t \leq 6$.

Answer: .....

**PART II**

Each problem is worth 12 points.

**Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers. CIRCLE YOUR ANSWER!**

## Problem 1

Evaluate the following indefinite integrals (clearly show the techniques of integration you use):

(a) $\int x\sqrt{x+1}\,dx$

(b) $\int \tan^{-1}(x+2)\,dx$

(c) $\int \frac{2}{x^2+4x+3}\,dx.$

## Problem 2

This problem has two separate questions. (Answer all the questions!)

(a) Find the **area** of the region bounded by the parabola $x = y^2 - 1$ and the slant line $x = y + 1$. (A sketch of the region might prove useful.)

(b) The region bounded by the curve $y = \sqrt{x}$ and the parabola $y = x^2$ is rotated about the vertical line $x = -1$. Find the **volume** of the solid obtained in this way. (A sketch might prove useful.)

## Problem 3

Write the function $f(x) = \dfrac{1}{1+x^4}$ as a power series, and then use your series to find the minimum number of terms needed to approximate the integral

$$\int_0^{1/10} \frac{1}{1+x^4}\,dx$$

with an error less than $10^{-9}$. (You do not need to compute and add up the terms in the sum!)

## Problem 4

Find the work done in pumping all the water out of a cubic container with edge 4 $m$ which is half full. The water has to be lifted all the way to the top of the cube before it can be removed. (You may use the approximation $g \approx 10\ m/s^2$ and the water density $\rho = 1,000\ kg/m^3$.)

## Problem 5

Two planes are given by the equations $x + y - z = 1$ for the plane $\mathcal{P}_1$, and $x - y - z = 1$ for the plane $\mathcal{P}_2$. Find parametric equations of the line of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

**DO NOT ENTER ANY PROBLEM SOLUTIONS OR WORK ON THIS PAGE.**

**Summary of scores on problems - for grading purposes only.**

| | Points |
|---|---|
| **Part I** | |
| Questions 1 – 10 | |
| **Part II** | |
| Problem 1 | |
| Problem 2 | |
| Problem 3 | |
| Problem 4 | |
| Problem 5 | |
| **Total Exam Score** | |

SCRATCH PAPER

(Scratch paper will not be graded)

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