EGR 265-6D, Math Tools for Engineering Problem Solving
December 11, 2015, 1:30pm to 4pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4 (incl. Bonus)	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Total (out of 100 + 5 Bonus)	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$yy' = -x, \quad y(1) = 1.$$

Problem 2 (10 points)

A liquid is heated to 150°F. It cools down according to Newton's law of cooling in a surrounding medium of temperature 70°F. The rate of cooling is $k = -0.1$.

(a) State the differential equation which governs the temperature of the medium at time t according to Newton's law of cooling.

(b) Solve this differential equation with the correct initial value (this can be done either as a separable or a linear equation).

(c) At what time has the temperature dropped to 75°F? (Logarithms do not need to be evaluated.)

Problem 3 (12 points)

Solve the initial value problem

$$y'' - 8y' + 16y = 16x - 4, \quad y(0) = 0,\ y'(0) = 3 \tag{1}$$

Problem 4 (12 points + 5 points bonus)

A 1 kg mass stretches a spring by 2 meter. The medium through which the mass moves offers a damping force with damping coefficient $\beta = 6$ kg/s. Include the correct units in all your answers below.
(a) Find the spring constant k, assuming that $g = 10$ m/s^2.

(b) Find the equation of motion of the mass if it is released from a position 20 cm below the equilibrium position with an upward velocity of 20 cm/s (choose the positive x-axis to be oriented downward).

(c) Is the system underdamped, critically damped, or overdamped?

(d) Will the mass return to the equilibrium position? If yes, when is the first time? If no, justify?

(e) (Bonus) An undamped spring with $m = 1$ kg and $k = 1\ \frac{\text{kg}}{\text{s}^2}$ has an added exterior driving force of the form

$$F(t) = \sin(0.9t)$$

or

$$F(t) = \sin(t)\,.$$

For both cases, without solving the DE, sketch what type of graph you expect for the solution $x(t)$ and name the physical phenomenon which is seen.

Problem 5 (10 points)

(a) Find the gradient of $f(x,y) = xy\cos(x) + y^3$.

(b) Evaluate the directional derivative of $f(x,y)$ at the point with coordinates $(0,1)$ in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

(c) Find the **direction** of steepest increase of $f(x,y)$ at the point $(0,1)$.

Problem 6 (8 points)

Find the parametric equations for the normal line to the graph of $z = 2x^3y - 5x$ through the point $(-1, -2, 9)$.

Problem 7 (8 points)

Find the work done by the force field $\mathbf{F}(x, y) = y^2\mathbf{i} - x\mathbf{j}$ when moving along the curve given by the graph of $y = \frac{1}{2}x^2$, $0 \leq x \leq 2$. Include the correct unit, assumed that force is measured in Newtons.

Problem 8 (12 points)

(a) Verify that the force field $\mathbf{F}(x, y) = (2xy + 2x)\mathbf{i} + (x^2 + 1)\mathbf{j}$ is conservative.

(b) Find a potential function $\phi(x, y)$ for $\mathbf{F}(x, y)$.

(c) Find the work done by the force field $\mathbf{F}(x, y)$ along the curve parameterized by $x = \cos(t)$, $y = \sin(2t)$, $0 \leq t \leq \pi/2$.

Problem 9 (10 points)

Let I_x and I_y denote the moments of inertia of a lamina with respect to rotation about the x-axis and y-axis, respectively.

(a) Find I_y for the triangular lamina of constant density $\rho(x, y) = 1$, bordered by the x-axis, the y-axis, and the graph of $y = 1 - \frac{1}{2}x$.

(b) Is $I_x > I_y$ or $I_x < I_y$? Give a reason for this which does not require to calculate I_x.

Problem 10 (10 points)

Let R be the region in the first quadrant, given by a quarter of the unit disk centered at the origin. Find

$$\iint_R \frac{y}{\sqrt{x^2+y^2}}\,dA$$