EGR 265, Math Tools for Engineering Problem Solving
October 26, 2015, 50 minutes

Name: .....

TEST II

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (15 points)

Find the general solution of

$$y'' - 4y' + 13y = 0$$

Problem 2 (20 points)

Find the general solution of

$$y'' + y' - 2y = 3e^{2x}$$

Problem 3 (25 points)

Solve the initial value problem

$$y'' - 4y = \cos x, \quad y(0) = 0, \quad y'(0) = 0$$

Problem 4 (20 points)

A mass of 2 kilograms stretches an undamped spring by 40 centimeters. For simplicity, use the approximate value $g = 10$ m/sec^2 for this problem.

(a) Find the value of the spring constant k using its correct metric unit.

(b) Find the angular frequency ω of free oscillations of the spring/mass-system.

(c) Find the equation of motion if the mass is released from the equilibrium position at a downward velocity of 20 cm/sec. Assume here that the positive x-direction is oriented downwards.

(d) Find the first time $t > 0$ at which the mass returns to the equilibrium position.

Problem 5 (8 points)

Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient β kg/sec.
(a) How should β be chosen to achieve critical damping?

(b) Which values of β make the system overdamped?

Problem 6 (12 points)

(a) Find the partial derivatives f_x and f_y of the function $f(x, y) = x^4y^3 + x^3 + y^2$

(b) Find the second order partial derivative g_{xy} of $g(x, y) = \cos(x^2 + y^2)$

Problem 7* (6 points bonus)

An undamped sping-mass-system consists of a metal ball of mass m and a spring of 'stiffness' k.
(a) Suppose you replace the ball by a heavier ball. Will the system oscillate slower and faster?
(b) Instead, keeping the same ball, you replace the spring by a stiffer spring. Will the system oscillate slower or faster?
Justify your answers!