EGR 265, Math Tools for Engineering Problem Solving
September 28, 2015, 50 minutes

TEST I

Name: .....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8*	
Total	

Problem 1 (4+4+4+4 Points)

Determine the order of the following ODEs. Also, state if they are linear or non-linear.

(a) $y''' + y' - xy = x$

(b) $y'' - y' = xy^2$

(c) $\dfrac{x}{y} = y'$

(d) $(xy')' = \cos x$

Problem 2 (6 Points)

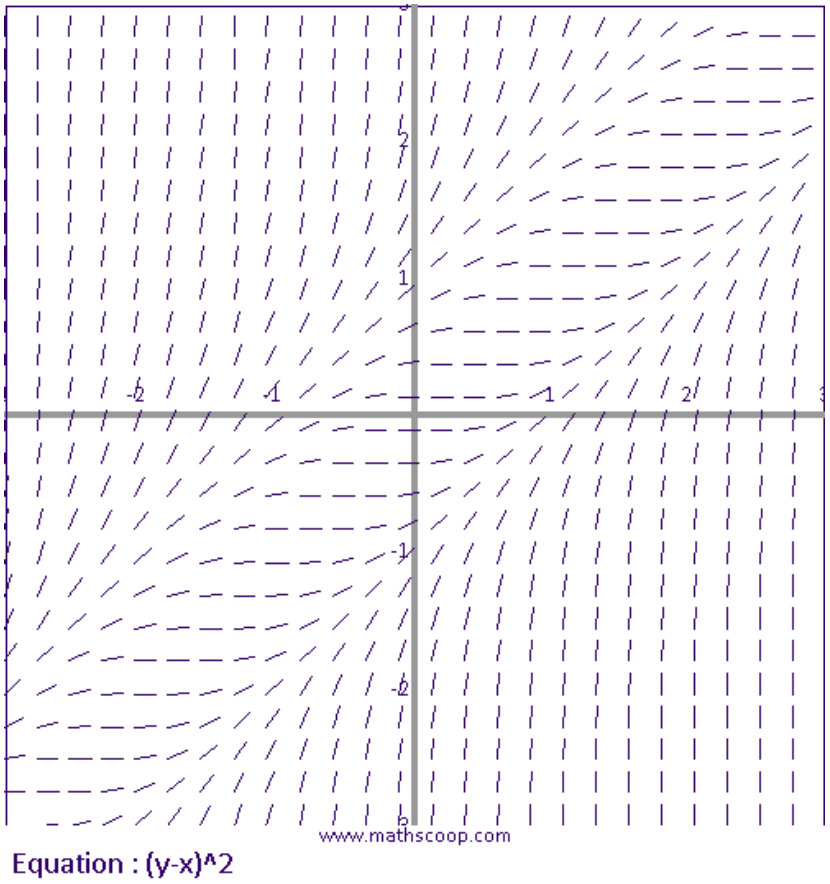
Find the values of the constant c for which the function $y = \frac{c}{x^2}$ is a solution of the differential equation

$$xy' + 3y = x^2y^2$$

Warning: This DE is neither separable nor linear, thus we don't have a method to systematically find its solutions.

Problem 3 (5+5 Points)

Below the direction field for $y' = (y - x)^2$ is given.



(a) Sketch the solution $y(x)$ of $y' = (y - x)^2$ with initial value $y(0) = 0$. (Note: Do not try to solve the DE! Our methods won't work.)

(b) Use the direction field to guess the solution of the IVP $y' = (y - x)^2$, $y(1) = 0$. Verify that your guess does indeed solve the IVP.

Problem 4 (16 Points)

Solve the IVP

$$y' - y^2 \cos x = 0, \quad y(\pi/2) = 1.$$

Problem 5 (16 Points)

Find the explicit solution of the IVP

$$xy' + y = xe^{x^2}, \quad y(1) = 0.$$

Problem 6 (16+5 Points)

(a) Find an implicit solution of the IVP

$$y' = \frac{x+2}{2y}, \quad y(0) = -2.$$

(b) What is the correct explicit solution of the above IVP? Justify your answer!

Problem 7 (15 Points)

The number of fish in a pond is given by $n(t)$, where the time t is measured in years. An initial population of $n(0) = 200$ fish grows at a constant rate $k = n'(t)/n(t)$. After 1 year 250 fish are present.

(a) Find the rate of growth k by solving the differential equation for $n(t)$ and using the given data.

(b) How much time passes until the population has doubled?

Note: Your answers will contain natural logarithms which do not need to be evaluated.

Problem 8* (5 Points Bonus)

Suppose that y_1 and y_2 are solutions of $y''+a_1(x)y'+a_0(x)y=0$. Circle those of the following functions which are guaranteed to also be solutions of the same DE.

(a) y_1+y_2, (b) y_1/y_2, (c) $5y_2$, (d) $(y_1)^2$

Which general principle is used here?