EGR 265, Math Tools for Engineering Problem Solving
November 18, 2015, 50 minutes

TEST III

Name: ....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (8+10 points)

(a) Find the gradient of the function $f(x, y) = x^2 y^2 + 2y^4$.

(b) Find the directional derivative of $f(x, y)$ at the point $(-2, \frac{1}{2})$ in the direction of the vector $\mathbf{v} = -4\mathbf{i} - 3\mathbf{j}$.

Problem 2 (10+4+4 points)

(a) For the function $g(x, y) = (x + 2y - 2)^{10}$ find the rate of steepest ascent at the point $(1, 1)$.

(b) Find a unit vector in the direction of steepest ascent for $g(x, y)$ at the point $(1, 1)$

(c) Find a unit vector in the direction of steepest descent for $g(x, y)$ at $(1, 1)$

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of $z = e^{3x+2y}$ at the point $(0, 0, 1)$.

(b) Also, find parametric equations for the normal line of $z = e^{3x+2y}$ at $(0, 0, 1)$.

Problem 4 (12 points)

Evaluate $\int_C 2x^3 \, ds$, where C is the graph of the function $y = \frac{1}{3}x^3$, $0 \le x \le 2$.

Problem 5 (14 points)

Find the work done by the force field $\mathbf{F}(x,y) = y^2 \, \mathbf{i} - x^2 \, \mathbf{j}$ along the curve parameterized by $x = t^3$, $y = t^2$, $0 \le t \le 1$. Include the correct unit, assuming that forces are measured in Newton (N).

Problem 6 (5+10+5 points)

(a) Verify that the given force field is conservative:

$$F(x, y) = (\sin y + y \cos x)\mathbf{i} + (\sin x + x \cos y)\mathbf{j}$$

(b) Find a potential for the force field from part (a).

(c) Find the work done by the force field from part (a) along the curve which consists of the three edges of the triangle with corners $(0, 0)$, $(1, 0)$, $(0, 1)$ and back to $(0, 0)$, moving counter-clockwise.

Problem 7* (6 points bonus)

Find a function $Q(x, y)$ such that the force field

$$\mathbf{F}(x, y) = 2xy\mathbf{i} + Q(x, y)\mathbf{j}$$

is conservative. What is the potential for the resulting force field?