EGR/MA265, Math Tools for Engineering Problem Solving
November 19, 2014

Name and section: _____
Instructors name: _____

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1. **Do not open this exam until you are told to do so.**
2. This exam has 8 pages including this cover. There are 8 questions, for a total of 100 points and 6 bonus points. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. **Organize your work,** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.
6. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.
8. **Turn off all cell phones**, and remove all headphones.

Do not write in the table below.

Question	Points	Bonus	Score
1	18	0	
2	18	0	
3	18	0	
4	12	0	
5	12	0	
6	10	0	
7	12	0	
8	0	6	
Total:	100	6	

1. (a) [9 points] Let $h(x, y, z) = (x^2 - 1)y^2 \sin(z)$. Find its third order partial derivative h_{xyz}.

(b) [9 points] Let $f(x, y) = \sin(xy + x^2)$. Find its partial derivatives f_x, f_y, and f_{yy}.

2. (a) [9 points] For the function $g(x, y) = 2\sqrt{2x - y}$, find a unit vector in the direction of steepest ascent at the point $P(2, 3)$. Also, find the rate of ascent in this direction.

(b) [9 points] Find the directional derivative of $g(x, y)$ at $P(2, 3)$ in the direction of $6\mathbf{i} - 8\mathbf{j}$.

3. (a) [12 points] Find an equation for the tangent plane to the graph of $z = (x-y)^2 - 2x$ at the point $(2, 1, -3)$.

(b) [6 points] Find parametric equations for the normal line of $z = (x-y)^2 - 2x$ at the point $(2, 1, -3)$.

4. [12 points] Evaluate $\int_C y\,ds$, where C is the graph of the function $x = y^2$, $0 \leq y \leq 1$.

5. [12 points] Find the work done by the force field

$$F(x, y) = e^x \mathbf{i} + (x + y^2)\mathbf{j}$$

along the curve traced by $x = \sin(t)$, $y = 2t$, $0 \leq t \leq \frac{\pi}{2}$.

6. Determine for each of the following force fields if it is conservative.
   (a) [5 points] $F(x, y) = (5 - 2xy)\,\mathbf{i} + (3x^2 - y^2)\,\mathbf{j}$
   (b) [5 points] $F(x, y) = (2ye^x - e^y)\,\mathbf{i} + (2e^x - xe^y)\,\mathbf{j}$

7. [12 points] For the conservative force field $F(x, y)$ from Problem 6 find a potential $\phi(x, y)$ and calculate the work done by this force field along the curve traced by the vector function $\mathbf{r}(t) = t^3\,\mathbf{i} + \sqrt{1 + 3t^2}\,\mathbf{j}$, $0 \le t \le 1$.

8. [6 points (bonus)] Give an example of a gradient (or conservative) force field $\mathbf{F}(x, y)$ (but not from Problem 6). Justify!