MA 125 CALCULUS I

Final Exam, December 10, 2014

Name (Print last name first): .....

Show all your work, justify and simplify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible but you don't need to compute numbers: $e^6 \sin(12/5) + 8$ is a fine answer.

All problems in Part I are 4 points each.

1. Use **the definition** of the derivative to show that the derivative of the function $y = f(x) = x^2$ is $f'(x) = 2x$.

2. Find the derivative $f'(x)$ if $f(x) = x^2 \sin(x)$.

3. Find the derivative $f'(x)$ if $f(x) = \ln(x^3 + x^2 + 1)$.

4. Find the derivative $f'(x)$ if $f(x) = \frac{x^3+1}{x^3-1}$.

5. Find the anti-derivative $\int x^2(1+\sqrt{x})\,dx$.

6. Find the anti-derivative $\int \sin^6(x)\cos(x)\,dx$.

7. Find the anti-derivative $\int x^3\sqrt{x^4+5}\,dx$.

8. Solve $\ln(x^2 + 1) = 5$.

9. If $F(x) = \int_2^x \sin(t^2 + 1)\, dt$, find $F'(x)$.

10. If oil leaks from a well at the rate of e^{-5t} (m^3/s), how much oil will leak in the first minute? (If you use your calculator to compute it is OK if you give an approximate answer.)

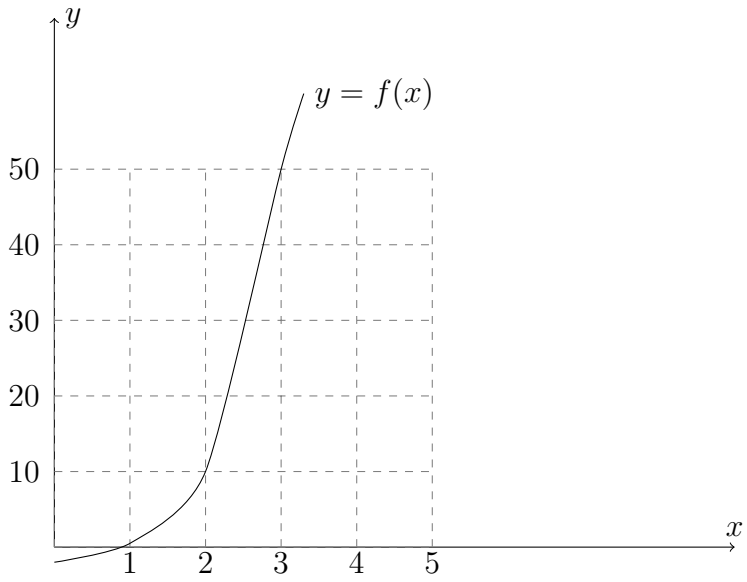
11. Approximate $\int_1^4 \frac{1}{x}\,dx$ using a Riemann sum with $n = 3$ terms and the midpoint rule. What does this number have to do with $\ln(4)$?

12. The velocity of a particle is given by $v(t) = t^2 + 1$ (m/s) .

    (a) Find the acceleration $a(2)$ of the particle,

    (b) How far does the particle travel in the first 5 seconds?

13. Given the graph of the function $f(x)$ below answer the following questions.



(a) Is $f(x)$ one-to-one? Explain!!

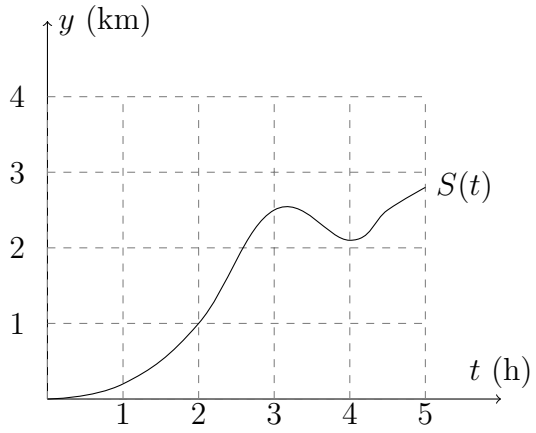
(b) Use the graph to approximate $f^{-1}(20)$.

(c) Use the graph to approximate $(f^{-1})'(20)$.

PART II

1. **9 points.** Find all local/absloute maxima/minima of the function $f(x) = (2x+1)^3(1-x)^5$ on the real line $(-\infty, \infty)$.

2. **9 points.** Let $S(t)$ be the function which specifies the distance (in km) from a runner to the start line at time t (in hours) of a race. The graph of $S(t)$ is given below:



Use the graph to give approximate answers to the following problems.

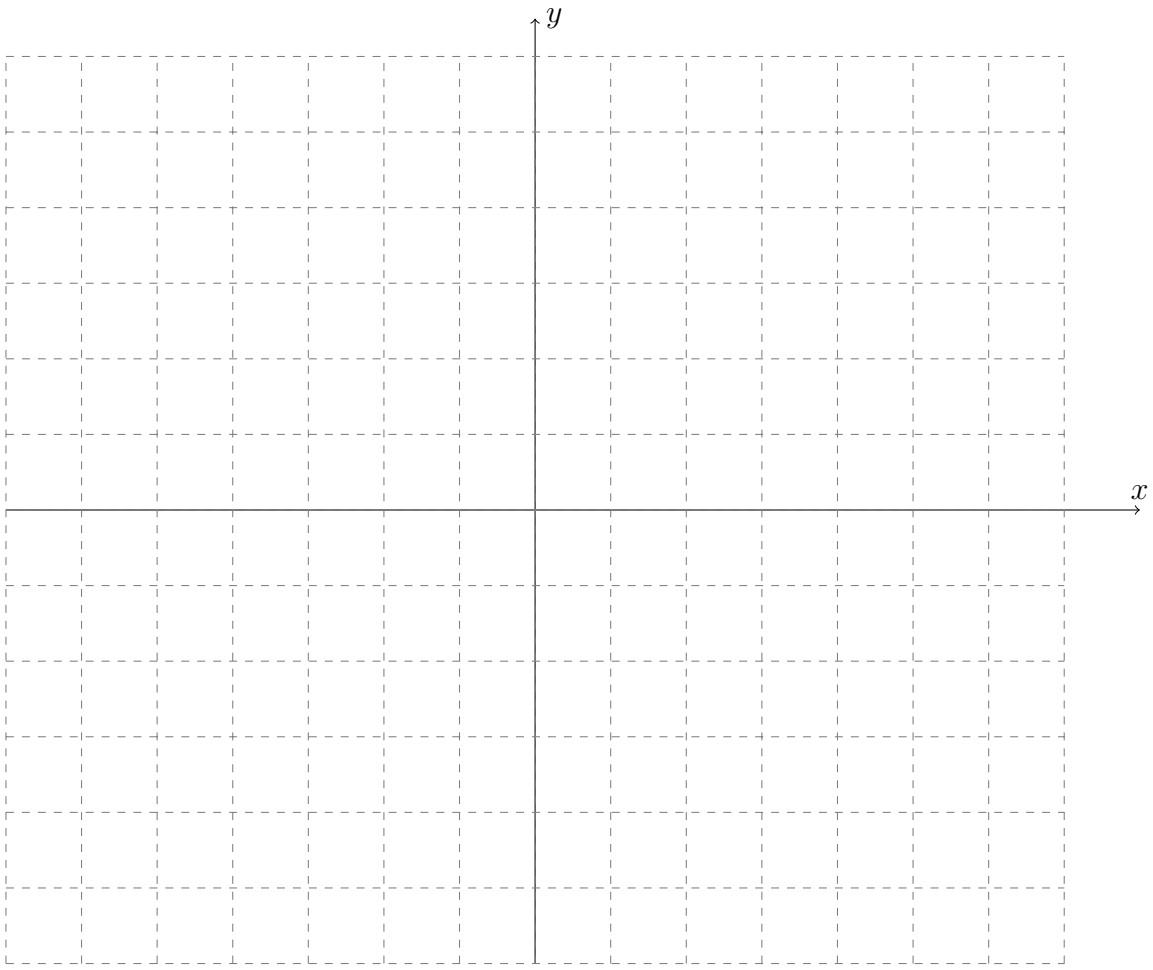
(a) When was the runner running the fastest?

(b) What happened between times 3 and 4?

(c) What is the meaning of $S^{-1}(1)$?

3. **12 points.** Graph the function $y = f(x) = \dfrac{x^2}{x^2 - 1}$. Find x and y-intercepts, horizontal and vertical asymptotes, all critical numbers, intervals of in-/de-creasing, local/absolute max/min

   Draw your graph on the next page.


y
x

4. **9 points.** Evaluate $\displaystyle\int_0^1 \frac{x^2}{(5-x)}\,dx$.

5. **9 points.** Find the dimensions of a can (i.e. a cylinder) of radius r, height h and volume $1\,(m^3)$ with minimal surface area. [Hint: the volume $V = \pi r^2 h$ and the surface area $S = 2\pi rh + 2\pi r^2$.]

Scratch paper