MA 125 CV, CALCULUS I

October 15, 2014

Name (Print last name first): .....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the absolute maximum and minimum of the function $y = f(x) = (x-5)^2(x+2)$ on the interval $[-3, 1]$.

2. Find the number c which satisfies the conclusion of the Mean Value Theorem for the function $y = f(x) = x^2 + x$ on the interval $[0, 2]$.

3. Find all critical numbers of the function $y = f(x) = \sqrt[3]{x^2 + x}$ and identify all local/absolute max/min if any.

4. Suppose that the **derivative** of a function $y = f(x)$ is:
$f'(x) = x^2 - x - 6$.

   (a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

   (b) At which x is the function $y = f(x)$ most rapidly decreasing?

   (c) What can you say about a formula for $f(x)$?

PART II

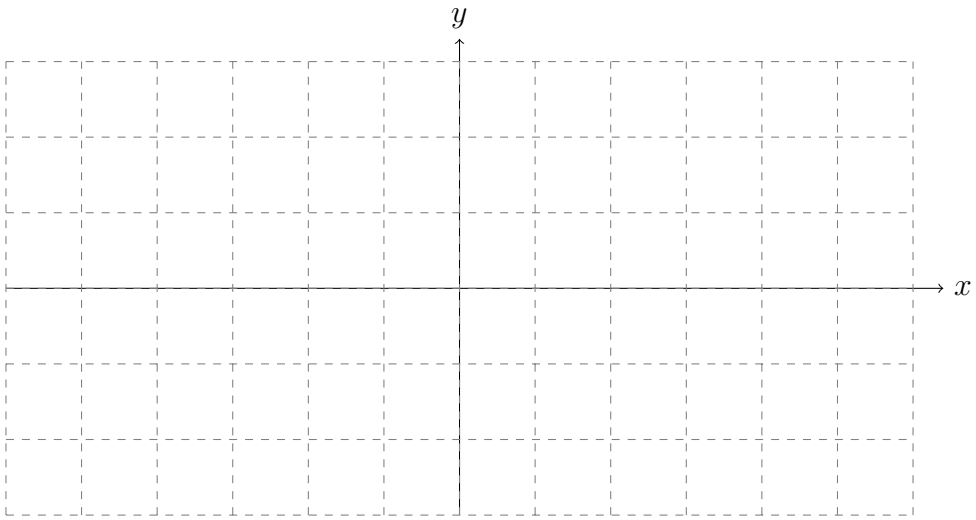
5. [**15 points**] The concentration of an average student during a 3 hour test at time t is given by $C(t) = 2t^3 - 3t^2 - 12t + 20$. When, during the test, is the student's concentration maximal?

6. [**15 points**] An oil refinery is located on the shore and an oil well is located 10 km off shore 30 km east of the refinery. [Hence if the refinery is located at $(0, 0)$ and the x-axis is the shore line, then the well is located at $(30, 10)$.] If it costs 10 million per mile to lay a pipe line in the ocean and 1 million per mile to lay a pipe line on land, how should one lay the pipe line from the well to the refinery to minimize the cost?

7. [**20 points**] **Use calculus** to graph the function $y = f(x) = \dfrac{x}{x^3 - 1}$. Indicate

   - x and y intercepts,
   - vertical and horizontal asymptotes (if any),
   - in/de-creasing; local/absolute max/min (if any).

   You must show work to justify your graph and conclusions. You can use decimal numbers to plot points (but mark them with exact values).



8. This question has two parts.

   (a) [**6 points**] Show that the equation $y = f(x) = 10x^3 + 10x + \sin(x) + \frac{1}{100} = 0$ has **exactly** one solution.

   (b) [**4 points**] Use linearization at an appropriate point to find an approximate solution of this equation.