MA 126 - 8C CALCULUS II

October 28, 2014

Name (Print last name first): .....

Student Signature: .....

TEST III

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. Limited partial credit is awarded for this part of the test!

Each question is worth 8 points.

Question 1

Evaluate the (indefinite) integral $\int \frac{2x}{x-4}\,dx$.

Answer: .....

Question 2

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{\sqrt[3]{x^5}}\,dx$$

Answer: .....

Question 3

Find the **area** of the region bounded by the curve $y = x^4$, the vertical line $x = 0$ and the horizontal line $y = 16$.

Answer: .....

Question 4

Find the **volume** of the solid obtained by rotating the region above the curve $y = x^2$ about the y-axis for $0 \leq x \leq 3$.

Answer: .....

Question 5

A (variable) force given by $f(x) = 2x$ Newtons moves a tiny object on a straight line from a distance of 1 m to a distance of 3 m. Find the **work** done.

Answer: .....

Question 6

Find the **length of the arc** of the circular helix with vector equation
$\mathbf{r}(t) = \langle\, 2\cos(t),\, \sqrt{5}\, t,\, 2\sin(t) \,\rangle$ when $0 \leq t \leq 1$.

Answer: .....

PART II

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

(a) Evaluate the (indefinite) integral

$$\int \frac{5x}{(x-2)(x^2+1)}\,dx.$$

(b) Determine whether the (improper) integral

$$\int_0^\infty xe^{-x}\,dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

Problem 2

(a) Find the **area** of the region enclosed by the parabola $y = 4x - x^2$ and the line $y = 2x$. (Hint: Sketching the region might prove useful here!)

(b) The region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the horizontal line $y = 2$. Find the **volume** of the solid obtained in this way.

Problem 3

(a) Find the **volume** of the solid obtained by rotating about the y-axis the region bounded by the curve $y = x^2$, the horizontal lines $y = 1$ and $y = 4$, and the vertical line $x = 0$.

(b) Find the **length of the arc** of the circular helix with vector equation $\mathbf{r}(t) = \langle\, 4\cos(t),\, 3t,\, 4\sin(t) \,\rangle$ when $0 \le t \le 1$.

Problem 4

This problem has two separate questions. (Answer each question!)

(a) Find the distance traveled by an ant crawling along the curve $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$ when $\pi/2 \leq t \leq \pi$.

(b) A rectangular swimming pool 3 m long, 1 m wide, and 1 m deep is full of water. Find the **work** needed to pump all the water out over the side. (Use the fact that the density of the water is 1,000 kg/m^3 and $g \approx 10$ m/s^2.)

SCRATCH PAPER

(Scratch paper will not be graded!)

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