EGR 265, Math Tools for Engineering Problem Solving
September 30, 2013, 50 minutes

TEST I

Name: .....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Total	

Problem 1 (16 Points)

Determine the order of the following ODEs. Also, state if they are linear or non-linear.

(a) $2xy' = e^y$

(b) $y^{(5)} - \cos(x)y'' = e^x y$

(c) $y'' + y^2 = y$

(d) $\dfrac{y''}{y' + y} = \cos(x)$

Problem 2 (11 Points)

Suppose $y = Cx^2$ is a solution to the DE $-\frac{5}{2}x^2y'' + \frac{1}{4}(y')^2 + 6x^2 = 0$.

(a) Find all the numerical values of C for which $y = Cx^2$ is a solution to the above ODE.

(b) Find a solution of the DE satisfying the initial condition $y(1) = 2$.

Problem 3 (15 Points)

Solve the IVP

$$y' + 2x(y-1)^2 = 0, \quad y(0) = 0.$$

Problem 4 (15 Points)

Solve the IVP

$$\frac{y'}{x} = 2e^{x^2}e^{-y}, \quad y(0) = 0.$$

Problem 5 (15 Points)

Solve the IVP

$$y' + 2y = xe^{-2x}, \quad y(0) = 1.$$

Problem 6 (14 Points)

The number of fish in a pond is given by $n(t)$, where the time t is measured in years. An initial population of $n(0) = 200$ fish grows at a constant rate $k = \frac{n'(t)}{n(t)}$. After 1 year 250 fish are present.

(a) Find the rate of growth k by solving the differential equation for $n(t)$.

(b) Find an expression for the time t_d necessary for the number of fish to double :

$$n(t_d) = 400.$$

Note : Your answers will contain natural logarithms which do not need to be evaluated.

Problem 7 (14 Points)

You want to chill a bottle of coke in the refrigerator. Suppose the initial temperature of the coke is 70°F and the refrigerator is set to be 35°F. After 5 minutes the coke has chilled down to 60°F.

Note: Your answers to the questions below will contain natural logarithms which do not need to be evaluated.

(a) Newton's Law of Cooling can be used to describe this process. Write down the corresponding IVP using an unknown cooling rate k.

(b) Solve the IVP and determine k by using information provided in the problem.

(c) When does the temperature of the coke reach 40°F?