Department of Mathematics UAB
MA227 Calculus 3
Final Exam
December 11, 2013

Each problem worth 10 points

1. Consider the equation

$$x^2 + y^2 + z^2 - 6x + 2y - 6z + 15 = 0.$$

Convert this equation to standard form, and identify the associated surface.

2. Find an equation for the surface consisting of all points that are equidistant from the point $(0, 0, -1)$ and the plane $z = 1$. Identify the surface.

3. Use differentials to estimate the amount of metal in a closed cylindrical can that is 20 cm high and 5 cm in diameter if the metal in the top and the bottom is 0.2 cm thick and the metal in the sides is 0.05 cm thick.

4. Find the local maximum and minimum values and saddle point(s) of the function

$$f(x, y) = x^3 + 6xy + 8y^3.$$

5. Find the absolute maximum and minimum values of $f(x, y) = xy^2 - 1$ on the set $D = \{(x, y) : x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 4\}$.

6. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 4x + 3y$ subject to the given constraint $x^2 + y^2 = 1$.

7. Use a double integral to find the area of the region inside the circle $(x-4)^2 + y^2 = 16$ and the circle $x^2 + y^2 = 16$.

8. Use a triple integral to find the volume of the tetrahedron bounded by the three coordinate planes and the plane $x + 2y + z = 1$.

9. By making an appropriate change of variables, evaluate the integral

$$\iint_D \cos(7\frac{y-x}{y+x})\, dA$$

where D is the trapezoidal region with vertices $(4,0)$, $(5,0)$, $(0,5)$, and $(0,4)$.

10. Use spherical coordinates to find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $\phi = \pi/4$.