MA-227-6D: Calculus III

Test#1, October 14, 2013

11 points each

1. Find a parametric equation for the tangent line to the curve

$$\vec{r}(t) = \langle t+3,\, 5t^2+14t+26,\, 2t\rangle$$

at the point $P(1, 18, -4)$.

2. Find the tangential and normal components of the acceleration vector for the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + 3t\vec{k}$ at the generic point $\vec{r}(t)$.

3. Find the limit, if exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 3y^4}{x^2 + y^2}.$$

4. Find $f_{rss}$ and $f_{rst}$ for $f(r, s, t) = r\ln(rs^2t^3)$.

5. Use differentials to estimate the amount of thin in a closed thin can with diameter 8 cm and height 12 cm if the thin is 0.04 cm thick.

6. Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ if

$$xyz = \sin(x^2 + y^2 + z^2).$$

7. Let $z = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$. Find $\partial z/\partial u$, $\partial z/\partial v$, and $\partial z/\partial w$ when $u = 2$, $v = 1$, $w = 0$.

8. Find the maximum rate of change of the function $f(x, y, z) = \tan(x + 2y + 3z)$ at $(-5, 1, 1)$ and the direction in which it occurs.

9. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

10. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse $E$. Find the points on $E$ that are nearest to and farthest from the origin.