EGR 265-6D, Math Tools for Engineering Problem Solving
December 14, 2012, 1:30pm to 4:00pm

Name (Print last name first): .....

Student ID Number: ....

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Bonus Problem	
Problem 9	
Problem 10	
Total	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$y' = ye^{2x}, \quad y(0) = 1.$$

Problem 2 (10 points)

Iron, which has a melting point of 2200°F, is heated in a furnace to 2500°F. After extraction from the furnace it cools down according to Newton's law of cooling in a surrounding medium of temperature 100°F. After two hours the Iron has cooled to 2300°F.
(a) Write down the IVP governing the cooling process using an unknown cooling rate k.
(b) Solve the IVP and determine k by using information provided in the problem.
(c) How long does it take the Iron to solidify?
Note: In parts (b) and (c) logarithms do not need to be evaluated.

Problem 3 (12 points)

Consider the second order differential equation

$$y'' - 4y' + 5y = 2e^{-x}. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Find the general solution of the inhomogeneous equation (1).

Problem 4 (12 points)

An 8 pound weight stretches an undamped spring by 2 feet. Assume that $g = 32$ ft/s^2. Include the correct units in all your answers below.
(a) Find the spring constant k and its correct unit.
(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.
(c) Find the particular solution of the equation if the mass is released from rest at a position of six inches above the equilibrium.
(d) What is the first positive time at which the mass returns to the equilibrium position?

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = \frac{1}{x+y^2}$.
(b) Evaluate the directional derivative of $f(x, y)$ at the point with coordinates $(0, 1)$ in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.
(c) Find a unit vector in the direction of steepest increase of $f(x, y)$ at the point $(0, 1)$.

Problem 6 (8 points)

Find the equation for the tangent plane to the graph of $z = ye^{x-y}$ at the point $(1, 1, 1)$.

Problem 7 (10 points)

Find the line integral

$$\int_C 2\,dx + y\sqrt{1+2x}dy,$$

where the curve C is given by the graph of $x = \frac{1}{2}y^2$, $0 \leq y \leq \sqrt{3}$.

Problem 8 (12 points)

(a) Determine for each of the following force fields if it is conservative.

(i) $\mathbf{F}(x, y) = xy\mathbf{i} - xy\mathbf{j}$

(ii) $\mathbf{F}(x, y) = (e^y + ye^x)\mathbf{i} + (xe^y + e^x + 1)\mathbf{j}$

(b) For the conservative force field $\mathbf{F}(x, y)$ from part (a) find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve $x = t^3$, $y = 1 - t^2$, $0 \leq t \leq 1$.

Bonus question (8 points)

(c) For the non-conservative force field $\mathbf{F}(x, y)$ from part (a), use Green's Theorem to find the work done along the curve C, where C is the positively oriented triangle with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$.

Problem 9 (10 points)

A lamina of constant density $\rho(x, y) = 1$ is bounded by the curves $y = x^2$, $x = 0$, and $y = 1$.
(a) Find the moment of inertia I_y with respect to the y-axis.
(b) Find the moment of inertia I_x with respect to the x-axis.

Problem 10 (8 points)

Rewrite the function $f(x, y) = \frac{y}{\sqrt{x^2+y^2}}$ using polar coordinates and find its integral over the quarter disk of radius 1 in the first quadrant.