MA227-6D, CALCULUS III

September 19, 2012

Name (Print last name first): ....

Student Signature: ....

TEST I

Each question is worth 10 points. Show all of your work for full credit.

Question 1. Find $\mathbf{i} \times \mathbf{j}$ and $\mathbf{i} \cdot (\mathbf{j} \times \mathbf{k})$. Give a geometric interpretation of both quantities.

Answer: ....

Question 2. Find parametric equations of the tangent line to the curve $\mathbf{r}(t) = (\cos(\pi t), t, \ln t)$ at $t = 1$.

Answer: ....

Question 3. Find the curvature of the curve $\mathbf{r}(t) = (\cos t, t, \ln t)$.

Answer: ....

Question 4. A particle moves along the curve $\mathbf{r}(t) = (t^3, t, e^t)$. Find its velocity, acceleration, speed, and the tangential and normal components of the acceleration at $t = 0$.

Answer: ....

Question 5. A particle moves with acceleration $\mathbf{a}(t) = (6t, \sin t, e^t)$. Find its position $\mathbf{r}(t)$ if the initial velocity and position are $\mathbf{v}(0) = (0, -1, 1)$ and $\mathbf{r}(0) = (1, -1, 2)$, respectively.

Answer: ....

Question 6. Find the arc length of the curve $\mathbf{r}(t) = (3t, 4t^{3/2}, 3t^2)$ with $0 \le t \le 1$.

Answer: ....

Question 7. Find the area of the triangle PQR with $P(2, 1, 5)$, $Q(-1, 3, 4)$ and $R(3, 0, 6)$.

Answer: ....

Question 8. Find the osculating plane of the curve $\mathbf{r}(t) = (t, \sin t, \cos t)$ at $t = 0$.

Answer: ....

Question 9. Let a be a constant. Show that the curve $\mathbf{r}(t) = (t, a\sin t, a\cos t)$ has constant curvature.

Answer: ....

Question 10. Find the angle between the curve $\mathbf{r}(t) = (t, \sin t, \cos t)$, and the curve $y^2 + z^2 = 1$, $x = 0$; at their intersection.

Answer: ....