MA 126 - ??   CALCULUS II
September ??, 2012

Name (Print last name first): ....

Student Signature: ....

$\boxed{\textbf{TEST I}}$

$\boxed{\textbf{Closed book - No calculators!}}$

$\boxed{\textbf{PART I}}$

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Find the equation of the sphere with center $(1, 0, -1)$ that passes through the point $(1, 2, -1)$.

Answer: ....

Question 2

Find the angle between the vectors $\mathbf{u} = \ <0, 0, 1>$ and $\mathbf{v} = \ <\sqrt{3}, 0, 1>$. Write your angle in degrees or radians!

Answer: ....

Question 3

Find the symmetric equations of the line through the point $(1, 1, -2)$ and parallel to the vector $\mathbf{u} = \; < 1, -1, 1 >$.

Answer: ....

Question 4

Find an equation of the plane that contains both the point $P(1, 1, 1)$ and the vectors $\mathbf{u} = \langle 0, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, 1, 0 \rangle$.

Answer: ....

Question 5

For what (numerical) value(s) of b are the vectors $\mathbf{u} = \langle b, 2, b^2 \rangle$ and $\mathbf{v} = \langle -1, b, 1 \rangle$ orthogonal?

Answer: ....

Question 6

Find the point of intersection of the plane $x + y - z = -1$ and the line given by the parametric equations

$$\ell_1 := \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + t \end{cases}$$

Answer: ....

$$\boxed{\textbf{PART II}}$$

Each problem is worth 14 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - <u>no credit for unsubstantiated answers</u>!

<u>Problem 1</u>

This problem has two separate questions (a) and (b). Answer each question.

(a) A constant force with vector representation $\mathbf{F} = 6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ moves an object along a straight line from the point $P(1, 1, 2)$ to the point $Q(3, 1, 3)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

(b) A women runs due north on the deck of a ship at 3 mph while the ship is moving east at a speed of 4 mph. Find the speed of the woman relative to the surface of the water. (We assume that the speed of the water is negligible compared with the speed at which the ship is moving.)

Problem 2

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(1,0,0)$, $Q(0,-2,0)$ and $R(0,0,3)$.

(b) Find the volume of the box generated by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Problem 3

Consider the two lines given by the parametric equations

$$\ell_1 = \begin{cases} x = 2t \\ y = -2 + 2t \\ z = 4 - t \end{cases} \quad \text{and } \ell_2 = \begin{cases} x = 10s \\ y = 3 + 5s \\ z = -3 + 2s \end{cases}$$

(a) Determine whether they are parallel.

(b) Determine whether they intersect. If they do intersect, find the point of intersection.

(c) Determine whether they are skew.

Problem 4

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the parametric equations of the line of intersection of the planes

$$x + 2y + 2z = 1 \quad \text{and} \quad x - 2y + 3z = 1.$$

(b) Find the distance from the point $P(3, 2, 1)$ to the plane $x = 0$.

Problem 5

A particle is traveling along the space-curve

$$\mathbf{r}(t) = \langle 4\cos(t),\ 2\sin(t),\ t\ \rangle$$

when $t \geq 0$.

(a) Determine the velocity vector of the particle at the time $t = \pi$.

(b) Find the parametric equations of the tangent line to this space-curve when $t = \pi$.

SCRATCH PAPER

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