Final Exam
Calculus I; Fall 2012
Part I

Part I consists of 10 questions, each worth 5 points. Clearly show all your work for each of the problems listed.

In 1-4, find y' if:

(1) $y = x^4 \tan(x)$

(2) $y = \frac{e^x}{x^2+1}$

(3) $y = (\sin(x))^{30}$

(4) $y = \cos(x^5 + x)$

(5) Find the critical numbers of $y = f(x) = (x - 1)^2(x + 1)^4$.

(6) Find all local/absolute maxima/minima of the function $y = 2x^4 + 8x$. Make sure to state both x and y values.

(7) Find all asymptotes of the function $y = \frac{1-x^2}{x^2+x-6}$

(8) Find all x-values where $y = \arctan(x^2)$ is **increasing**.

(9) Find the most general form for the **anti-**derivative of
$y = \frac{x^5-x}{\sqrt{x}}$

(10) Use calculus to find the dimensions of a rectangle whose area is 5 and whose perimeter is minimal.

Part II

Part II consists of **6** problems; the number of points for each part are indicated by [x pts]. You **must** show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [**10 pts**] Use implicit differentiation to find the derivative if
$$y^2 = \sin(xy)$$

(2) [**6 pts**] Find the linearization of the function $y = f(x) = \sin(x)$ at $a = \pi$.

(3) **[4 pts]** Use the linearization in problem 2 to estimate $\sin(\pi - \frac{1}{10})$

(4) Note that you are not asked to determine where the function is concave up/down nor do you need to find the points of inflection. **Be careful when computing f'(x)!**

Given the function $y = f(x) = \frac{(x+2)^2}{x^2-1}$

   (a) **[2 pts]** Find the x and y intercepts of the function.

   (b) **[2 points]** Find all asymptotes.

(c) [**2 pts**] Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) [**2 pts**] Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinates of each of them).

(e) [**5 pts**] Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **absolute/local maxima/minima**.

(5) **[9 pts]** If $y = f(x) = \sqrt[3]{x^2 - 1}$, find the absolute maximum and minimum of $f(x)$ on the closed interval $[-2, 2]$. (Include the appropriate y values, simplify when possible.)

(6) [**10 pts**] A cannon tracks an air plane which flies at a *constant* altitude of 5 km and a speed of 300 km/h directly toward the cannon. How fast (in radians/hour) is the angle between the cannon and a vertical line decreasing when the plane is 5 km away from the point P straight above the gun which is at an altitude of 5 km.