MA 125, CALCULUS I

November 28, 2012

Name (Print last name first): ....

Student Signature: ....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Part I consists of 6 questions. Each question is worth 7 points.
Clearly write your answer in the space provided after each question.

Question 1

Find the absolute minimum and absolute maximum values of the function

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

on the closed interval $[0, 2]$. (Be sure to give both x and y-coordinates!)

Question 2

Use the Mean Value Theorem to show that the equation

$$x^5 + x - 1 = 0$$

has **exactly one** solution in the interval $[-1, 1]$.

Question 3

Find the open interval on which the function

$$g(x) = x^2 e^x$$

is decreasing. (Clearly indicate the end-points of your interval!)

Question 4

Find the interval(s) on which the function

$$h(x) = \frac{1}{20}x^5 - \frac{1}{6}x^3$$

is concave up. (For an extra credit, sketch the graph.)

Question 5

Find the most general antiderivative of the function $f(x) = e^x - x^2 + \frac{1}{x} + \sin x$.

Question 6

Find two positive numbers whose sum is 20 and whose product is **maximal**. (Just a guess is not enough. Do the relevant calculus work.)

PART II - Problem Solving Skills

Points for each problem are indicated

Part II consists of 4 problems. You must show your work to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 [12 points]

Suppose that the derivative of a function f is given by

$$f'(x) = (x-2)^3(x+1)$$

Answer all the following questions. (For an extra credit, find a formula for the function $f(x)$).

(a) Find all the critical numbers of the function f.

(b) On what interval(s) is the function f increasing? (Justify your answer!)

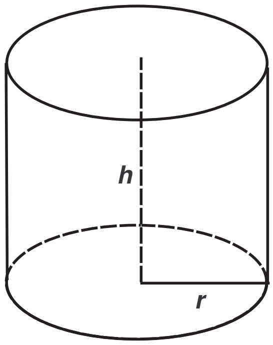
(c) On what interval(s) is the function f decreasing? (Justify your answer!)

Problem 2 [12 points]

A manufacturer plans to produce a cylindrical container with an open top. Let r denote the radius of the base and h denote the height of the cylinder. The bottom has area πr^2 and the side wall has surface area $2\pi rh$. The volume of the container is $\pi r^2 h$.

Suppose the volume must be equal to 1000π (cubic meters).

Find r and h so that the area of the bottom and side wall combined is minimal.



Volume = $\pi r^2 h$

Bottom = πr^2

Side wall = $2\pi rh$

Problem 3 [12 points]

An object moves along a straight line with acceleration

$$a(t) = 12t - 4.$$

Use antiderivatives to answer the following questions.

(a) Find the velocity function $v(t)$ of the object if its initial velocity is $v(0) = 3$.

(b) Find the position function $s(t)$ of the object if its initial position is $s(0) = 0$.

Problem 4 [22 points]

Consider the function f given by

$$f(x) = \frac{x^2}{x^2 - 4}.$$

Answer all the following questions.

(a) Find the x and y-intercept(s) of the curve.

(b) Find, if any, the vertical and horizontal asymptote(s) of the curve.

(c) Find the interval(s) of increase, and the interval(s) of decrease.

(d) Find, if any, all local maximum and local minimum value(s). [Be sure to give both x and y-coordinates!]

(e) Find interval(s) where the function is concave down, and interval(s) where it is concave up. [Hint: Factoring out might prove useful in your calculations!]

(f) Find inflection points (if any).

(g) Use the information from parts (a)–(f) above to sketch the graph.