MA 126-7B, CALCULUS II

November 03, 2010

Name (Print last name first): .....

Student Signature: .....

TEST III

Closed book - No calculators are permitted!

PART I

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{\sqrt[3]{x^2}}\,dx$$

Answer: .....

Question 2

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{x^{5/4}}\,dx.$$

Answer: .....

Question 3

Find the **area** of the region bounded by the line $y = x + 1$, the x-axis, the y-axis and the vertical line $x = 1$.

Answer: .....

Question 4

Find the **area** of the region bounded by the parabola $y = x^2$ and the horizontal line $y = 1$.

Answer: .....

Question 5

Find the **volume** of the solid obtained by rotating the curve $y = \sqrt{x}$ about the x-axis for $0 \leq x \leq 4$.

Answer: .....

Question 6

Find the length of the arc of the circular helix with vector equation
$\mathbf{r}(t) = \langle \cos(t), \sin(t), \sqrt{3}\, t \rangle$ when $0 \leq t \leq 1$.

Answer: .....

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) Determine whether the (improper) integral

$$\int_1^\infty \frac{1}{x\sqrt{\ln(x)}}\,dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

(b) Determine whether the (improper) integral

$$\int_1^\infty 2xe^{-x^2}\,dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

Problem 2

(a) Find the **area** of the region enclosed by the parabola $y = 5x - x^2$ and the line $y = x$. (Hint: Sketching the region might prove useful here!)

(b) Find the **area** of the region enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Problem 3

(a) Find the **volume** of the solid obtained by rotating about the y-axis the region bounded by the curve $y = x^3$, the horizontal lines $y = 1$ and $y = 8$, and the vertical line $x = 0$.

(b) The region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the vertical line $x = -1$. Find the **volume** of the solid obtained in this way.

Problem 4

(a) Find the **volume** of the solid obtained by rotating about the y-axis the region **above** the curve $y = \sqrt[3]{x}$ when $0 \leq y \leq 1$.

(b) Find the **length of the arc** of the curve $y = \dfrac{2}{3}\, x^{3/2}$ when $0 \leq x \leq 1$.

Problem 5

This problem has two separate questions. (Answer each question!)

(a) Find the **length of the arc** of the circular helix with vector equation $\mathbf{r}(t) = \langle\, 4t,\, 3\cos(t),\, 3\sin(t) \,\rangle$ when $-1 \leq t \leq 1$.

(b) A rectangular swimming pool 2 m long, 1 m wide, and 1 m deep is full of water. Find the **work** needed to pump all the water out over the side. (Use the fact that the density of the water is 1,000 kg/m^3 and $g \approx 10$ m/s^2.)

SCRATCH PAPER

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