Instructor:_____ Name:_____

Final Exam

Calculus I; Fall 2010

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

In 1-4, find y' if:

(1) $y = \frac{\sin(x)}{x^3+1}$

(2) $y = (x^3 + x^2 + x + 1)^{30}$

(3) $\lim_{x \to 0^+} x \ln(x)$

(4) $y = \sin(x^5 + x)$

(5) Find the critical points of $y = f(x) = x^2(x+1)^3$

(6) Find all local maxima/minima of the function $y = 4x^3 - 6x^2$. Make sure to state both x and y values.

(7) Find the vertical and horizontal asymptotes of the function $y = \frac{5x^2-7}{x(x^2-4)}$

(8) Find the interval(s) in the x-axis where $y = xe^x$ is **decreasing**

(9) Find the most general form for the **anti-**derivative of $y = \frac{x^5-x^2+\sqrt{x}}{x^2}$

(10) **Use calculus** to find the dimensions of a rectangle with perimeter 10 and maximal area.

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) **[10 pts]** Use implicit differentiation to find the derivative y' if $\sqrt{x^2+y^2} = xy$

(2) **[5 pts]** Find the linearization of the function $y = f(x) = \sin(x)$ at $a = \pi$.

(3) **[3 pts]** Use the linearization in problem **??** to estimate $\sin(\pi + \frac{1}{10})$

(4) Given the function $y = f(x) = \frac{x^2}{x^2-1}$

(a) **[2 pts]** Find the x and y intercepts of the function.

(b) **[3 pts]** Find the vertical and horizontal asymptotes of the function.

(c) **[2 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

(d) **[2 pts]** Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(e) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(f) **[1 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point when possible).

(g) **[5 pts]** Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **x,y-intercepts, local maxima/minima and point(s) of inflection.**

(5) **[5 pts]** If $y = 2x\sqrt{x+1}$ find the absolute max and min on the interval $-1 \leq x \leq 3$. (Include the appropriate y values of the maximum and minimum.)

(6) **[10 pts]** If a rock falls from the top of a 30m tall building with an initial velocity of 5m/sec (downward), find equations for the velocity and positions at time t. Use these to find the velocity of the rock when it hits the ground. [You may use that the gravitational acceleration $g \approx 10 m/sec^2$.]

Scratch paper