EGR 265, Math Tools for Engineering Problem Solving
November 16, 2009, 50 minutes

Name: .....

TEST III

Problem 1 (9+9 points)

(a) Let $f(x, y) = 2y^3 - 6x^2y$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x, y) = x\cos(xy)$ find g_x, g_y and g_{xy}.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = \ln(x^2 + y^4)$ find its direction **and** rate of steepest descent at the point $P(2, 1)$.

(b) Find the directional derivative of $h(x, y)$ at $P(2, 1)$ in the direction of the vector from $P(2, 1)$ to $Q(5, 3)$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the level surface $3x^2 + y^4 + 2z^2 = 15$ at the point $(2, 1, 1)$.

(b) Also, find parametric equations for the normal line of $3x^2+y^4+2z^2 = 15$ at $(2, 1, 1)$.

Problem 4 (12 points)

Evaluate $\int_C x\,ds$, where C is the graph of the function $y = x^2$, $0 \leq x \leq 2$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = 2xy\mathbf{i} + ye^y\mathbf{j}$$

along the curve C parameterized by $x = t^3$, $y = t$, $0 \leq t \leq 1$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = (3x^2 + y)\mathbf{i} + (x - 2y)\mathbf{j}$

(b) $F(x, y) = xe^{2y}\mathbf{i} - x^2e^{2y}\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = t\sin(\pi t)\mathbf{i} + t\cos(\pi t)\mathbf{j}$, $0 \leq t \leq 1/2$.