

**EGR 265, Math Tools for Engineering Problem Solving**  
September 16, 2009, 50 minutes

Name (Print last name first): .....

Student ID Number: ..... .....

<b>TEST I</b>
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Problem 1

Determine the order of the following ODEs. Also, state if they are linear or non-linear.  
(4P+4P+4P+4P)

(a)  $yy'' = \cos x$

(b)  $y^{(5)} - y^4 = e^x y$

(c)  $\frac{y - \cos x}{y'} = e^x$

(d)  $y' + \cos y = x$

Problem 2

(a) Which of the following functions are solutions of  $x^4 y' + 2xy^2 = 4x^5$ ? (8P)

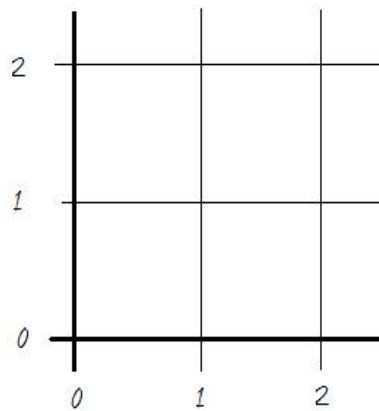
$$y_1 = x^2, \quad y_2 = x, \quad y_3 = -x^2, \quad y_4 = -2x^2.$$

(b) Which of the functions from part (a) solve the initial value problem  $x^4y' + 2xy^2 = 4x^5$ ,  $y(0) = 0$ ? (4P)

(c)\* (Bonus) Does your answer to part (b) agree with the content of the Existence and Uniqueness Theorem for first order ODEs? If yes, why? If no, why not? (5P\*)

Problem 3

(a) In the  $3 \times 3$ -grid of points  $x = 0, 1, 2$  and  $y = 0, 1, 2$  provided in the figure below draw a direction field for  $y' = x(y - 1)$ . (8P)



(b) Without solving the DE, use the direction field to read off the solution of the IVP  $y' = x(y - 1)$ ,  $y(1) = 1$ . (4P)

Problem 4

Solve the IVP (15P)

$$y' = x(y - 1), \quad y(2) = 2.$$

Problem 5

Solve the IVP (15P)

$$\frac{y'}{x} + 2y = 1, \quad y(0) = 1$$

Problem 6

Solve the IVP (15P)

$$y' - y^2 \cos x = 0, \quad y\left(\frac{\pi}{2}\right) = 1$$

Problem 7

A cup of milk is chilled to  $40^{\circ}\text{F}$  in the refrigerator and then taken out into a room of  $75^{\circ}\text{F}$ . After 5 minutes it has warmed up to  $50^{\circ}\text{F}$ .

Note: Your answers to the questions below will contain natural logarithms which do not need to be evaluated.

(a) Newton's Law of Cooling can also be used to describe warming processes as in this problem. Write down the corresponding IVP using an unknown warming rate  $k$ . (4P)

(b) Solve the IVP and determine  $k$  by using information provided in the problem. (8P)

(c) When does the temperature of the milk reach  $70^{\circ}\text{F}$ ? (3P)