FALL 2009 — MA 227 — FINAL

Name: ______________________________

1. Part I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Find the dot product (or scalar product) of the vectors $\langle 1, -3, 2\rangle$ and $\langle -2, 1, 3\rangle$.

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(2) Express the length of the curve $\boldsymbol{r}(t) = \cos(t)\boldsymbol{i} + 2\sin(t)\boldsymbol{j} + 3t\boldsymbol{k}$, $0 \leq t \leq 6\pi$ as an integral. (Do not evaluate the integral!)

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(3) Find the gradient of the function $f(x, y) = \sqrt{xy^2 + y}$.

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(4) Compute the Jacobian of the transformation $x = u^2 - 2v$, $y = 2u + v^2$.

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(5) Find rectangular coordinates of the point with spherical coordinates $\rho = 3$, $\phi = \pi$, $\theta = \pi/4$.

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(6) Find the linearization $L(x, y)$ of $f(x, y) = \ln(x + y^2)$ at the point $(1, 2)$.

(7) Reverse the order of integration in the iterated integral $\int_0^2 \int_{x^2}^4 f(x, y)\, dy\, dx$. (Do not evaluate the integral!)

(8) Find a potential function of the conservative vector field $\mathbf{F} = (1 + ye^{xy})\boldsymbol{i} + xe^{xy}\boldsymbol{j}$.

(9) Write down the iterated integral for $\iint_D 3y dA$ where D is the triangular region with vertices $(0, 0)$, $(1, 1)$ and $(2, 0)$. Do not evaluate the integral.

(10) Evaluate $\iint_D dA$ where $D = \{(x, y) : 0 \leq x^2 + y^2 \leq 4, 0 \leq y, 0 \leq x\}$.

2. Part II

There are 6 problems in Part II, each worth 10 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Find the distance between the lines (i.e., the distance between the closest points on these lines)

$$x = 2 + t, \quad y = 1, \quad z = 2 - t$$

and

$$x = 1 - s, \quad y = 1 + s, \quad z = 0.$$

(2) A ball is thrown from ground level at an angle of $\pi/4$ radians to the ground at a speed of v_0 meters per second. The path of the ball for time $t \geq 0$ is described by the vector function $\boldsymbol{r}(t) = \langle x(t), y(t) \rangle$, where the first component is along the horizontal while the second is along the vertical direction.

(a) What is the initial velocity vector $\boldsymbol{r}'(0)$?

(b) Assuming that gravity is the only force acting on the ball, and that the gravitational acceleration g is equal to 10 meters per second per second, find the acceleration vector $\boldsymbol{a}(t) = \boldsymbol{r}''(t)$ for the ball.

(c) Find the component functions $x(t)$ and $y(t)$ of the position vector $\boldsymbol{r}(t)$ in terms of the initial speed v_0.

(d) If the ball lands 20 meters away, find the time t_0 when the ball hits the ground, and the initial velocity v_0.

(3) Suppose that you are climbing a hill whose shape is given by the graph of the surface

$$z = 1000 - 0.005x^2 - 0.01y^2$$

where x, y, and z are measures in meters and you are standing at the point with coordinates $(20, -10, 997)$. The positive x-axis points east, the positive y-axis points north, and the positive z-axis points up.

(a) If you walk southeast will you start to ascend or descend? Find the rate of ascent, being careful to state the units involved.

(b) In which direction is the rate of increase largest? What is the rate of ascent in this direction?

(4) Find the points (x, y, z) on the double cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 4, 0)$.

(5) Find the volume of the solid lying between the planes $z = 0$, $z = x$ above the triangle bounded by the lines $x = 0$, $y = x$, and $y + x = 1$ in the x-y-plane.

(6) Use spherical coordinates to find the mass of the solid that lies within the sphere $x^2 + y^2 + z^2 = 1$ and above the cone $z = \sqrt{3(x^2 + y^2)}$ if the density μ of the material in the solid is given by

$$\mu(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}.$$