MA 126 - CALCULUS II
Friday, December 11, 2009

Name (Print last name first):. . . . . .

Signature: . . . . . .

Section: . . . . . Instructor Name: . . . . . .

FINAL EXAM

Closed Book. No calculators are permitted.

PART I

Each question is worth 4 points.

Part I consists of 10 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the exam. No partial credit is awarded for this part of the exam! CHECK YOUR ANSWERS!

Question 1

Find the angle between the vectors $\mathbf{u} =< 1, 1, 0 >$ and $\mathbf{v} =< 1, 0, 1 >$. (Give the angle in degrees or radians.)

Answer: . . . . . .

Question 2

Find the area of the parallelogram generated by the vectors $\mathbf{u} =< 0, 1, 1 >$ and $\mathbf{v} =< 1, 0, 1 >$. (Your answer must be a real number!)

Answer: . . . . . .

Question 3

Find the parametric equations of the line that passes through the point $P(2, 3, 1)$ and is parallel to the vector $\mathbf{v} = \langle 3, 1, -7 \rangle$.

Answer: .....

Question 4

Find an equation of the plane that passes through the point $P(0, 2, -1)$ and is perpendicular to the vector $\mathbf{n} = \langle 1, 2, 3 \rangle$.

Answer: .....

Question 5

Use the Fundamental Theorem of Calculus to find the derivative of the function $g(x) = \int_1^x \sin(t^3)\, dt$.

Answer: .....

Question 6

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{\sqrt[5]{x^2}}\,dx$$

Answer: .....

Question 7

Find the area of the region bounded by the parabola $y = 3x^2 + 5$, the horizontal line $y = 0$, and the vertical lines $x = 0$ and $x = 1$. (Your answer must be a number!)

Answer: .....

Question 8

Evaluate the indefinite integral $\int \cos^3(x)\,dx$.

Answer: .....

Question 9

Evaluate the indefinite integral $\int \frac{x}{x+x^3}\,dx$.

Answer: .....

Question 10

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{3/5}}$ is divergent, absolutely convergent, or conditionally convergent.

Answer: .....

PART II

Each problem is worth 12 points.

Part II consists of 5 problems. You must show your work on this part of the exam to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit. CIRCLE YOUR ANSWER!

Problem 1

Two planes are given by the equations $x + y - z = 3$ for the plane $\mathcal{P}_1$ and $x - y + z = 1$ for the plane $\mathcal{P}_2$.

(a) Find the coordinates of a point of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

(b) Find the normal vector (i.e., the vector perpendicular) to the plane $\mathcal{P}_1$ and the normal vector to the plane $\mathcal{P}_2$.

(c) Find the parametric equations of the line of intersection of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

Problem 2

This problem has two separate questions. (Answer all the questions!)

(a) Find the length of the arc of the circular helix with vector equation $\mathbf{r}(t) = \langle 3\cos(t),\, 3\sin(t),\, 4t\rangle$ when $-1 \le t \le 5$.

(b) Determine whether the (improper) integral

$$\int_1^\infty \frac{\ln(x)}{x^2}\, dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

Problem 3

Evaluate the following integrals (clearly show the techniques of integration you use):

(a) $\displaystyle\int \frac{\sin(\sqrt{x})}{2\sqrt{x}}\,dx$

(b) $\displaystyle\int x^2\cos(x)\,dx$

(c) $\displaystyle\int \frac{x-4}{x^2+x-2}\,dx.$

Problem 4

This problem has two separate questions. (Answer all the questions!)

(a) Find the **area** of the region enclosed by the parabola $x = y^2 - 3y$ and the parabola $x = y - y^2$.

(b) The region enclosed by the curve $y = \sqrt{x}$ and the parabola $y = x^2$ and is rotated about the vertical line $x = -1$. Find the **volume** of the solid obtained in this way.

Problem 5

This problem has two separate questions. (Answer all the questions!)

(a) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt{n+1}}.$$

Be sure to check any endpoints that exist!

(b) Find the Maclaurin series for the function $f(x) = \sin(x)$, and use it to write out the Maclaurin series for the function $g(x) = \sin(x^2)$.

DO NOT ENTER ANY PROBLEM SOLUTIONS OR WORK ON THIS PAGE.

Summary of scores on problems - for grading purposes only.

	Points
Part I	
Questions 1 – 10	
Part II	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total Test Score	

SCRATCH PAPER

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