MA 126 - 7B CALCULUS II

September 14, 2009

Name (Print last name first): .....

Student Signature: .....

TEST I

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Find the equation of the sphere with center $(-1, 2, 3)$ that passes through the point $(2, 6, 3)$.

Answer: .....

Question 2

Find the angle between the vectors $\mathbf{u} =< 1, -1, 3 >$ and $\mathbf{v} =< -1, 3, -2 >$.

Answer: .....

Question 3

Find the (parametric) equations of the line through the point (1,2,3) and parallel to the vector $\mathbf{u} =< 2, 1, -1 >$.

Answer: .....

Question 4

Find an equation of the plane that passes through the point $P(0, 2, -1)$ and is perpendicular to the vector $\mathbf{n} = \langle 1, 2, 3 \rangle$.

Answer: .....

Question 5

Are the vectors $\mathbf{u} =< -1, 1, -3 >$ and $\mathbf{v} =< 3, 0, -1 >$ orthogonal?

Answer: .....

Question 6

Find the area of the parallelogram generated by the vectors $\mathbf{u} =< 1, 1, 0 >$ and $\mathbf{v} =< 1, 0, 1 >$.

Answer: .....

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

This problem has two separate questions (a) and (b). Answer each question.

(a) A constant force $\mathbf{F} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ moves an object along the line with displacement vector $\mathbf{D} = -4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$. Find the work done if the (magnitude of the) displacement is measured in feet and the (magnitude of the) force is measured in pounds.

(b) A women runs due north on the deck of a ship at $3\sqrt{3}$ mph while the ship is moving east at a speed of 3 mph.

(1) Find the speed of the woman relative to the surface of the water.

(2) Find the direction of the woman relative to the surface of the water. (Express your answer as an angle with respect to specific direction(s) north, south, east, or west.)

Problem 2

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(1, 2, 1)$, $Q(1, 1, 1)$, and $R(1, 1, 2)$.

(b) Find the volume of the box generated by the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$ and $\mathbf{c} = \mathbf{j} + \mathbf{k}$.

Problem 3

Consider the two lines

$$\ell_1 = \begin{cases} x = 1 + 2t \\ y = 1 - t \\ z = -1 + 3t \end{cases} \quad \text{and } \ell_2 = \begin{cases} x = 1 - 2s \\ y = 1 + 2s \\ z = 2 - s \end{cases}$$

(a) Determine whether they are parallel.

(b) Determine whether they intersect. If they do intersect, find the point of intersection.

(c) Determine whether they are skew lines.

Problem 4

This problem has two separate questions (a) and (b). Answer each question.

(a) Find symmetric equations of the line of intersection of the planes

$$4x + y + z = 1 \quad \text{and} \quad x + y - z = 3.$$

(b) Find the equation of the plane containing the points $(0, 0, 0)$, $(1, 2, 3)$ and $(-1, -1, 1)$.

Problem 5

An particle is traveling along the space-curve

$$\mathbf{r}(t) = \left\langle \frac{t}{1+t}, t^3 - 1,\ te^t \right\rangle$$

when $t > -1$.

(a) Determine the velocity vector of the particle at each time t.

(b) Find the unit tangent vector $\mathbf{T}(0)$ at time $t = 0$.

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