EGR 265, Math Tools for Engineering Problem Solving
October 15, 2008, 50 minutes

Name (Print last name first): ....

 Student ID Number: ....

$$\boxed{\textbf{TEST II}}$$

Problem 1 (20 points)

Solve the initial value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 6$$

Problem 2 (20 points)

Solve the initial value problem

$$y'' - 3y' - 4y = 5, \quad y(0) = -\frac{1}{4}, \quad y'(0) = 0$$

Problem 3 (20 points)

Use Variation of Parameters to find the general solution of

$$y'' - 2y' + y = x^2 e^x.$$

(Recall the formulas

$$u_1' = \frac{\begin{vmatrix} 0 & y_2 \\ f & y_2' \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}, \quad u_2' = \frac{\begin{vmatrix} y_1 & 0 \\ y_1' & f \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}$$

used in variation of parameters.)

Problem 4 (30 points)

A mass of 2 kilograms stretches an undamped spring by 2 meters and 45 centimeters.
(a) Find the equation of motion if the mass is released 50 cm below the equilibrium position at a downward velocity of 1 m/s. Assume here that the positive x-direction is oriented downwards.
(b) Write the equation of motion in the form $x(t) = A\sin(\omega t + \phi)$ and determine A and ϕ.
(c) Find the first positive time at which the mass passes through the equilibrium position.

Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is numerically equal to b times the instantaneous velocity, where b is a constant. How does b have to be chosen for the system to become critically damped?