MA 126 CALCULUS I FALL 2008

FINAL EXAM

Name (Print last name first):………………………………………………………….

Instructor: …………………………………. Section: …………………………..

Part 1

Each question is worth 4 points.

Part I consists of 11 questions. Place your answer on the answer-line next to the question. Space is provided between questions for you to work each problem. Little partial credit will be awarded on Part 1 problems, so work carefully.

1 Evaluate $\int \cos x \sin^2 x\, dx$ ____________________

2. Determine whether the series $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ converges or diverges. ____________________

3. Evaluate $\int_1^{\infty} \frac{1}{1+x^2}\, dx$ or show it diverges. ____________________

4. Evaluate $\int \tan^2 x \, dx$ ____________________

5. Find the cosine of the angle between the vectors $\vec{u} = 3\vec{i} + 2\vec{j}$ and $\vec{v} = 5\vec{j} + \vec{k}$. ____________________

6. Find the length of the curve $x = \frac{2}{3}(y-1)^{3/2}$, $1 \le y \le 5$. ____________________

7. Find the parametric equations of the line through the two points $P(3,-1,1)$ and $Q(-2,1,1)$. ____________________

8. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}3}{5^n}$. __________________

9. A crate is hauled 8 meters up a ramp under a constant force of 200 newtons applied at an angle of 30^0 to the ramp. Find the work done. __________________

10. Determine whether $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{e^n} \right)$ is absolutely convergent, conditionally convergent, or divergent. __________________

11. If $f(x) = \int_1^x \sqrt{4-t^3}\, dt$, find $f'(x)$. __________________

PART II

Each problem is worth 8 points

Part II consists of 7 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

Problem 1

(a) Find the equation of the plane passing through the three points $P(2,1,-1)$, $Q(0,3,2)$, and $R(1,0,-2)$.

(b) Find the area of the triangle with vertices $P, Q,$ and R where P, Q, R are given in part (a).

Problem 2

(a) Find a power series representation for $f(x) = \dfrac{1}{1+x}$ in the form $\sum_{n=0}^{\infty} c_n x^n$. On what interval of x-values does the series equal the function? Explain or show work that justifies your choice of interval.

(b) Use your answer from (a) to find a power series representation for $g(x) = \dfrac{1}{(1+x)^2}$.

Problem 3

Evaluate and simplify.

(a) $\int \frac{3x^3 - 5x + 8}{x^3 + 4x}\, dx$

(b) $\int_0^{2\ln 3} e^{-x/2}\, dx$

Problem 4

Find the area under the graph of $y = x^2 e^{-x}$ and above the x-axis from $x = 0$ to $x = 1$.

Problem 5

Consider the region in the *xy* plane bounded by $y = x^3$, $y = 0$, and $x = 2$. Set up an integral, **but do not evaluate it**, for the volume of the solid generated by rotating this region about each line given below. In each case your integral should be complete with limits and in terms of just one variable.

(a) $y = 0$

(b) $x = 0$

Problem 6

A rectangular water tank has base 10 ft by 15 ft and is 12 ft deep. It contains water that is 8 ft deep. Find out how much work is required to pump the water over the rim. (Assume that the density of water is 62.5 lbs/ft^3.)

Problem 7

Find a power series representation for $f(x) = xe^{-x^3}$. Use your series to approximate $\int_0^{1/10} xe^{-x^3}\,dx$ with an error less than 10^{-8}.