MA 126   NAME_____

Closed Book, No Calculators   Fall, 2008

PART 1. **Part 1 consists of 6 questions. Do your work and clearly write your answer in the space provided. No partial credit is awarded for this part of the test. (5 points each)**

1. Determine whether the geometric **series** $\sum_{1}^{\infty} \dfrac{2}{5^n}$ is convergent or divergent. If it is convergent, find

    its sum.

    Answer: _____

2. Determine whether the **series** $\sum_{k=1}^{\infty} \dfrac{-n}{2n+1}$ is convergent or divergent. If it is convergent find its sum.

    Answer: _____

3. Determine whether the **sequence** given by $a_n = \dfrac{n}{\ln n}$ converges or diverges. If it is convergent, find

    its limit.

    Answer:_____

4. Find the radius of convergence of the **series** $\sum\limits_{n=0}^{\infty} \dfrac{x^n}{n!}$.

Answer:_____

5. If the **series** $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^2}$ were approximated by its third partial sum, what would be the maximum error to expect?

Answer:_____

6. Use the Integral Test to determine if the **series** $\sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln n}$ converges or diverges.

Answer:_____

<u>Part 2.</u> **Part 2 consists of 5 problems worth 14 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.**

Problem 1

(a) Determine whether the series $\sum_{n=0}^{\infty} \dfrac{n}{n^3 + 2}$ is convergent or divergent. (You must justify your answer.)

(b) Consider the alternating series given by $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{\sqrt{n}}$. Show how you use appropriate series tests to determine whether this series is absolutely convergent, conditionally convergent, or divergent. You must clearly state your final conclusion and provide justification for it.

Problem 2

Find the radius of convergence and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(x-1)^n}{n(3^n)}$.

Be sure to check any endpoints that exist.

Problem 3

(a) Find a power series representation for the function $f(x) = \dfrac{1}{1+x^2}$. Then state the interval on which the series equals the function.

(b) Use the series in (a) to find a power series representation for $f(x) = \arctan x = \tan^{-1} x$.

Problem 4

Determine whether or not each of the following **sequences** $\{ a_n \}$ converges. Give reasons.

a. $a_n = \cos n\pi$

b. $a_n = \dfrac{n^2}{2^n}$

c. $a_n = \cos\left(\dfrac{3}{n}\right)$

d. $a_n = \dfrac{\cos n}{\sqrt{n}}$

Problem 5

a. Determine whether the sequence $\{a_n\} = \left\{n + \dfrac{1}{n}\right\}$ is increasing, decreasing, or not monotonic.

b. If $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ is convergent when $x = -3$ and divergent when $x = 7$, is :

(a) $\displaystyle\sum_{n=0}^{\infty} c_n (-9)^n$ convergent?   (b) $\displaystyle\sum_{n=0}^{\infty} c_n$ convergent?   Give reasons for your answers.