

PART 1. Part 1 consists of 6 questions. Do your work and clearly write your answer in the space provided. No partial credit is awarded for this part of the test. (5 points each)

1. Set up, **but do not evaluate**, an integral for the **area** of the region bounded by $y = x^2$ and $y = 4$.

Answer: _____

2. Evaluate the improper integral, or show it diverges. $\int_0^{\infty} xe^{-x^2} dx$

Answer: _____

3. A spring of natural length 1.5 ft. requires a force of 10 lbs. to hold it stretched 0.5 ft past its natural length. How much work would need to be done to stretch the spring from its natural length to an additional 1.5 ft.?

Answer: _____

4. The integral $\int_2^{\infty} \frac{1}{x^2 - 2x} dx$ is improper for two reasons. State the reasons below. **Do not evaluate** the integral.

5. Is the **area** under the curve $y = \frac{1}{\sqrt{x}}$ and above the x-axis and between the lines $x = 0$ to $x = 1$ finite or infinite? If finite, what is it?

Answer: _____

6. Find the **length** of the helix given by $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + t\vec{k}$, $0 \leq t \leq \pi$.

Answer: _____

Part 2. Part 2 consists of 5 problems worth 14 points apiece. Show all your work for full credit! Displaying only the final answer (even if correct) without the relevant steps is not enough.

Problem 1

Find the **area** of the region between the parabola $y^2 = 4x$ and the line $4x - 3y = 4$.

Problem 2

Use the method of slicing (disk method) to set up integrals, complete with limits of integration and in terms of just one variable, to find the **volume** of the solid generated by revolving the region bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the following lines. **Do not evaluate the integrals.**

a. the x-axis

b. $y = 2$

Problem 3

Sketch the region bounded by $y = x^3$, $x = 2$, and the x -axis. Use cylindrical shells to find the **volume** of the solid of revolution generated by revolving this region about the y -axis.

Problem 4

a. Find the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{3}$.

b. Use the Comparison Theorem to determine whether or not $\int_1^{\infty} \frac{1}{\sqrt{x^2 - 1}} dx$ converges or diverges.

Problem 5

Find the **work** done in pumping the water out of a full circular cone of height $h = 7m$ and radius $r = 5m$. You may use the fact that the density of water is $1000 \text{ kg}/m^3$ and that the acceleration due to gravity is $9.8m/sec^2$. You can **just set up** the needed integral in terms of one variable and the appropriate limits. **You do not need to evaluate the integral.**