

MA 125 CALCULUS I FALL 2008

December 05, 2008

FINAL EXAM

Name (Print last name first):.....

Student Signature: .....

Instructor: ..... Section: .....

**PART I**

Part I consists of 10 questions. Place your answer on the answer-line next to the question. Space is provided between questions for you to work each question (if you wish). No partial credit is awarded on Part I problems, only your entry on the answer line will be graded.

Each question is worth 4 points.

Question 1

Evaluate  $\lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x - 5}$ .

Answer: .....

Question 2

Evaluate  $\lim_{x \rightarrow 0} \frac{x^2 + \sin(x)}{x}$ .

Answer: .....

Question 3

Find  $y'$  if  $y = \frac{x-1}{x+1}$ .

Answer: .....

Question 4

Find the value of  $x$  for which the curve  $y = (x-2)e^{-x}$  has a horizontal tangent line.

Answer: .....

Question 5

Find an equation of the tangent line to the curve  $y = 1 - \ln(x+1)$  at the point  $(0, 1)$ .

Answer: .....

Question 6

Find  $y'$  if  $y = \sqrt[3]{x^2 + x - 1}$ .

Answer: .....

Question 7

Find all the critical numbers of the function  $f(x) = x^4 - 2x^2$ .

Answer: .....

Question 8

Find the open interval(s) on which the function  $f(t) = \frac{1}{3}t^3 - 4t + 5$  is decreasing.

Answer: .....

Question 9

Find all inflection points of the curve  $y = \frac{3}{10}x^5 - x^3$ . [Be sure to give the  $x$  and the  $y$  coordinates of each point!]

Answer: .....

Question 10

Find the most general antiderivative of  $f(x) = \cos(x) - \sec^2(x) + 3$  on the interval  $(-\pi/2, \pi/2)$ .

Answer: .....

**PART II**

Each problem is worth 10 points.

**Part II consists of 6 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.**

**Problem 1**

Consider the equation

$$x^3 + x \sin(y) + xy + 1 = 0$$

in which  $y$  is implicitly defined as a function of  $x$ .

- (a) Use implicit differentiation to find  $y'$ .
- (b) Is the curve  $x^3 + x \sin(y) + xy + 1 = 0$  rising or falling at the point  $(-1, 0)$ ? (Justify your answer!)
- (c) Find an equation of the tangent line to the curve  $x^3 + x \sin(y) + xy + 1 = 0$  at the point  $(-1, 0)$ .

**Problem 2**

Consider the function

$$f(x) = x^3 - 3x^2 + 1.$$

- (a) Find each open interval where  $f(x)$  is increasing (you should find two intervals in all), and the open interval where it is decreasing.
- (b) Find all local maximum and minimum points of  $f(x)$ . [Be sure to give the  $x$  and the  $y$  coordinate of each point!]
- (c) Find the open interval where  $f(x)$  is concave down, and the open interval where it is concave up.
- (d) Find the inflection point of  $f(x)$ . [Be sure to give the  $x$  and the  $y$  coordinate!]
- (e) Sketch a graph of  $f(x) = x^3 - 3x^2 + 1$ . (Clearly indicating the relevant items above on your graph.)

**Problem 3**

Consider the function

$$f(x) = \sqrt[5]{1-x}.$$

- (a) Find the linearization (or linear approximation) of  $f(x)$  at  $a = 0$ .
- (b) Use the linearization of  $f(x)$  to find an approximation of  $\sqrt[5]{1.1}$ . (Show your answer to 2 decimal places!)
- (c) Another way to find an approximation of  $\sqrt[5]{1.1}$  is to use Newton's method to find a root of the equation  $x^5 - 1.1 = 0$ .  
Use Newton's method with initial approximation  $x_1 = 1$  to find  $x_2$ , the second approximation to the root of the equation

$$x^5 - 1.1 = 0.$$



**Problem 5**

The volume of a box with square base and open top is given by  $V = x^2h$ , where  $x$  is the length of each side of the square base and  $h$  is the height of the box. The surface area of the box is given by  $S = x^2 + 4xh$ . Assume that  $S = 27 \text{ m}^2$  of material is available to make the box.

- (a) Express the volume  $V$  as a function of  $x$  only by using the equation  $x^2 + 4xh = 27$ . (Simplify your answer!)

- (b) Use the above to find the dimensions of an open top box of maximal volume. (That is, find  $x$  and  $h$  such that the volume  $V$  is maximal on the interval  $x > 0$ .)



**Problem 6**

Use antiderivatives to answer the following questions.

- (a) Find  $f(x)$  for  $x > 0$  if it is known that  $f'(x) = \frac{x^2 - 2\sqrt{x} + 1}{x}$  and that  $f(1) = 1/2$ .

- (b) Find the antiderivative  $F(x)$  of the function

$$f(x) = \frac{1}{1+x^2} - 1,$$

given that

$$F(1) = \pi/4.$$

Summary of scores on problems - for grading purposes only. Do not enter any problem solutions or work on this page.

	Points
<b>Part I - Question 1</b>	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
<b>Part II - Problem 1</b>	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
<b>Total Test Score</b>	