MA 125 - DV, CALCULUS I
November 24, 2008

Name (Print last name first): .....

Student Signature: .....

TEST IV

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find all the critical numbers of the function $f(x) = \frac{1}{3}x^3 - 9x$.

Answer: .....

Question 2

The function $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-2, 2]$. Find all the numbers c that satisfy the conclusion of the Mean Value Theorem. (Hint: You should find three numbers in all!)

Answer: .....

Question 3

Find the absolute maximum value of the function $g(x) = 4x - x^2$ on the closed interval $[0, 1]$.

Answer: .....

Question 4

Find the open interval on which the function $g(x) = x^3 - 27x - 15$ is increasing.

Answer: .....

Question 5

Find the open interval on which the function $h(x) = xe^x$ is concave down.

Answer: .....

Question 6

Find the most general antiderivative $F(x)$ of the function $f(x) = 7 + e^x - \sin x$.

Answer: .....

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Suppose that the derivative of a function $f(x)$ is given by

$$f'(x) = (x-1)^4(x+1)^3(x-2)^2.$$

Answer all the following questions.

(a) Find all the critical numbers of the function $f(x)$.

(b) On what interval(s) is the function $f(x)$ increasing? (Justify your answer!)

(c) On what interval(s) is the function $f(x)$ decreasing? (Justify your answer!)

Problem 2

Consider the function

$$f(x) = \frac{1}{12}x^4 - \frac{1}{2}x^2 + 1.$$

Answer all the following questions.

(a) Find the (open) interval of increase, and all the (open) intervals of decrease.

(b) Find all local maximum and minimum points. [Be sure to give the x and y-coordinates of each point.]

(c) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up.

(d) Find the inflection points. [Be sure to give the x and the y coordinate!]

(e) Use the information from parts (a)–(d) to sketch the graph.

Problem 3

This problem has two separate questions. (Answer all the questions.)

(1) Find the dimensions of a rectangle with area 25 cm^2 whose perimeter is as small as possible. (Show your work!)

(2) Find a positive number such that the sum of the number and its reciprocal is as small as possible. (Show your work!)

Problem 4

This problem has two separate questions. (Answer all the questions.)

(a) Find the most general antiderivative of the function

$$f(x) = 2x + 5x^{2/3} + e^{-x} + \frac{3}{4}\sqrt[3]{x}.$$

(b) Find the most general antiderivative of the function

$$f(x) = \frac{x - x\sin x + x(1+x^2)^{-1}}{x}.$$

(Hint: Simplifying might prove useful!)

Problem 5

An object moves along a straight line with acceleration

$$a(t) = 10 + 6t - 12t^2.$$

(a) Find the velocity function $v(t)$ of the object if its initial velocity is $v(0) = 5$ mph.

(b) Find the position function $s(t)$ of the object if its initial position is $s(0) = 3$ mi.

SCRATCH PAPER

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