FALL 2007 — MA 227-6B — TEST 2
OCTOBER 15, 2007

Name: _____

1. Part I

There are 6 problems in Part 1, each worth 4 points. Place your answer on the line to the right of the question. Only your answer on the answer line will be graded.

(1) Describe or sketch the domain of the function $f(x, y) = \ln(6 - 3x^2 - 3y^2)$.

_____

(2) A bug crawls on a metal plate along curve given by $x(t) = t + 1$, $y(t) = t^2 + 2$. The temperature on the plate is given by $T(x, y, t) = 10 + xy + 2xt$. How warm is the spot the bug reaches at $t = 2$?

_____

(3) Find the first order partial derivatives of $f(x, y) = xy^2 - 2y/x$.

_____

(4) What is the direction of steepest ascent of the function $h(x, y) = xy$ at the point $P(2, 1)$?

_____

(5) The gradient of a function f is given by $\nabla f = \langle x - 2y^2, 4 - 2x \rangle$. Find all the critical points of f.

---

(6) Find the tangent plane to the sphere $x^2 + y^2 + z^2 = 1$ at the point $(0, 0, 1)$. Hint: Think, do not compute.

2. Part II

There are 3 problems in Part 2, each worth 12 points. On Part 2 problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Consider the function $f(x, y) = \sqrt{x^2 + 6y}$.
   (a) Compute $f(5, 4)$.
   (b) Find the linear approximation of f at the point $P(5, 4)$.
   (c) Use the linearization to approximate the value $f(5.2, 3.9)$.

(2) Use Lagrange multipliers to find the minimal and maximal values of $f(x, y) = 6x + y$ on the ellipse $3x^2 + y^2 = 26$. Where do they occur?

(3) (a) Identify which of the points $P(1,1)$, $Q(2,2)$, $R(3,3)$ are critical points of $f = (2x - x^2)(2y - y^2)$.
(b) Classify the critical points of f found in (a).
(c) Find all critical points of $g(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$.