MA 126-6A, CALCULUS II
November 14, 2007

Name (Print last name first): .....

Student ID Number (last four digits): ....

TEST III

No calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Evaluate the indefinite integral $\int \sin^9 x \cos x \, dx$.

Answer: .....

Question 2

Evaluate the indefinite integral $\int \frac{x}{x+1} \, dx$.

Answer: .....

Question 3

Determine whether the improper integral is convergent or divergent. Evaluate the integral if it is convergent.

$$\int_1^\infty \frac{1}{x^3}\,dx$$

Answer: .....

Question 4

Find the area of the region enclosed by the parabola $y = x^2$, and the lines $y = 0$ and $x = 1$.

Answer: .....

Question 5

Find the volume of the solid obtained by rotating the curve $y = x^3$ about the y-axis for $0 \le y \le 1$.

Answer: .....

Question 6

Find the length of the "arc" $y = 1 + (\sqrt{3})\,x$, when $0 \le x \le 1$. (Your answer must be a real number!)

Answer: .....

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) Evaluate the definite integral

$$\int_0^4 \sqrt{16 - x^2}\, dx.$$

(b) Evaluate the indefinite integral

$$\int \frac{x - 19}{x^2 - 8x - 9}\, dx.$$

Problem 2

(a) Determine whether the (improper) integral

$$\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$$

is convergent or divergent. Evaluate the integral if it is convergent.

(b) Find the exact area of the region between the graph of the function $f(x) = 2xe^{-x^2}$ and the x-axis when $0 \leq x < \infty$.

Problem 3

(a) Find the **area** of the region **enclosed** by the parabola $y = x^2$ and the line $y = x$.

(b) Find the numerical values of c such that the **area** of the region **bounded** by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is equal to 72. (You should find two values in all!)

Problem 4

(a) Find the **volume** of the solid obtained by rotating about the x-**axis** the region **bounded** by the curve $y = \sqrt{x+1}$, the horizontal line $y = 0$ and the vertical lines $x = 0$ and $x = 2$.

(b) Use the method of **cylindrical shells** to find the **volume** generated by rotating about the y-**axis** the region **bounded** by the curve $y = 2x^2 - x^3$ and the line $y = 0$.

Problem 5

(a) Find **the length of the curve** $y = \sqrt{4 - x^2}$ for $0 \leq x \leq 2$. (Hint: It might be helpful to notice that this arc is a portion of a well known graph in the plane!)

(b) Find **the length of the curve**

$$y = \frac{x^2}{2} - \frac{\ln x}{4}$$

when $1 \leq x \leq 3$.

SCRATCH PAPER

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