Instructor:________________________ Name:________________________

Final Exam

Calculus I; Fall 2007

Part I

Part I consists of 8 questions, each worth 5 points. Clearly show your work for each of the problems listed.

In 1-3, find y' if:

(1) $y = \frac{x^2-1}{x^2+1}$,

(2) $y = x\ln(x^2+1)$,

(3) $y = \sqrt[5]{\cos(x)}$.

(4) Find the equation of the tangent line to the graph of the function $y = f(x) = \sqrt{x}$ at $x = 4$,

(5) If the position at time t is given by $S(t) = \sin(5t)$, find the acceleration as a function of t.

(6) Find the most general anti-derivative of the function $y = f(x) = (7x + 3)^{21}$.

(7) Evaluate the limit

$$\lim_{x \to 0} \frac{x - \sin(x)}{x^3}$$

(8) Use calculus to find two positive numbers whose sum is 100 and whose product is maximal.

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) [**9 pts**] Find the absolute maximum value and the absolute minimum value of the function $f(x) = x^3 - 3x^2 + 9$ on the interval $[-1, 3]$.

(2) Consider the implicit function $y^3 + xy + x^3 = 11$.

(a) [**6 pts**] Use implicit differentiation to find y'.

(b) [**4 pts**] Find the equation of the tangent line to the curve $y^3 + xy + x^3 = 11$ at the point $(2, 1)$.

(3) An arrow is shot straight upward with an initial velocity (at time $t = 0$) of $v(0) = 15\ m/s$. [You may use that the acceleration function $a(t) = -10\ m/sec^2$.]

(a) [**3 pts**] Find an equation for the velocity function $v(t)$.

(b) [**3 pts**] Find an equation for the position function (you may assume that at time $t = 0$ the arrow is $S(0) = 0$ meters above the ground.

(c) [**3 pts**] Find the maximal height of the arrow.

(4) Consider the function $y = f(x) = x^4 - 4x^3$.

(a) **[2 pts]** Find the x and y intercepts of the function.

(b) **[2 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(c) **[2 pts]** Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(d) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(e) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).

(f) **[3 pts]** Use the above information to graph the function on the next page

(5) [**9 pts**] Find the linear approximation of the function $y = f(x) = 1/\sqrt{x}$ at $a = 9$ and use it to approximate the value of $f(9.1) = 1/\sqrt{9.1}$.

(6) The volume of a round cylindrical can of radius r and height h is given by $V = \pi r^2 h$ and its surface area is given by $S = 2\pi r^2 + 2\pi rh$. Assume that $S = 10cm^2$ of material is available.

(a) [**4 pts**] Express V as a function of r only by using the equation $S = 2\pi r^2 + 2\pi rh = 10$. [It is a good idea to simplify this expression before doing the next step.]

(b) [**6 pts**] Use the above to find the dimensions of a can of maximal volume (i.e., find r and h such that the volume V is maximal on the interval $r > 0$).