FALL 2006 — MA 227-8B — TEST 3

Name: ____

1. Part I

There are 4 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Evaluate $\int_{-1}^{1}\int_{0}^{3}(2x+y^2)\,dx\,dy$.

Answer: 20.

(2) Compute $\iint_D 5\,dA$ where D is the right triangle with legs 3 and 4.

Answer: 30.

(3) Compute the Jacobian of the transformation $x = u + 5v$, $y = 2u - 3v$.

Answer: -13.

(4) Express $\iint_D f(x,y)\,dA$ as an iterated integral, where D is the region bounded by the curves $x = 0$ and $x + y^2 = 4$. [Do not compute the integral!]

Answer: $\int_{-2}^{2}\int_{0}^{4-y^2} f(x,y)\,dx\,dy$.

A possible answer: $\int_{0}^{4}\int_{-\sqrt{4-x}}^{\sqrt{4-x}} f(x,y)\,dy\,dx$.

2. Part II

There are 3 problems in Part II, each worth 8 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Find the area of the surface $z = 2xy$ that lies within the cylinder $x^2 + y^2 = 4$.

$$\iint_D \sqrt{1 + (2y)^2 + (2x)^2}\, dA = \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2} r \, dr \, d\theta = \frac{\pi}{6} \left(17^{3/2} - 1\right).$$

(2) Evaluate the integral $\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy$ by reversing the order of integration. In addition, sketch the domain of integration.

$$\int_0^1 \int_0^x e^{-x^2}\,dy\,dx = \int_0^1 xe^{-x^2}\,dx = \frac{1}{2}\Big(1 - \frac{1}{e}\Big).$$

(3) Evaluate the triple integral $\iiint_E x\,dV$, where E is the tetrahedron bounded by planes $x = 0$, $y = 0$, $z = 0$, and $2x + y + z = 4$.

$$\begin{aligned} I &= \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} x\,dz\,dy\,dx \\ &= \int_0^2 \int_0^{4-2x} x(4-2x-y)\,dy\,dx \\ &= \int_0^2 \left[4xy - 2x^2y - xy^2/2\right]_0^{4-2x} dz \\ &= \int_0^2 8x - 8x^2 + 2x^3\,dx \\ &= \frac{8}{3}. \end{aligned}$$